Sample Proof: a(B+C)=aB+aC

Given mxn matrices B and C, and scalar a, prove a(B+C)=aB+aC.

Proof: To show that the matrices a(B+C) and aB+aC are equal, we must show they are the same size and that corresponding entries are equal.

Same size:
Since B and C are mxn, B+C is mxn thus a(B+C) is mxn also.
Since B is mxn, aB is mxn. Since C is mxn, aC is mxn. Thus the sum aB+aC is mxn.

ijth entry of a(B+C) = ijth entry of aB+aC

Long way: Show whole matrix. Very clear but a lot of writing.

If B= $\begin{bmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ b_{i1} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & & \vdots & & \vdots \\ b_{m1} & \cdots & b_{mj} & \cdots & b_{mn} \end{bmatrix}$ and C= $\begin{bmatrix} c_{11} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & & \vdots & & \vdots \\ c_{i1} & \cdots & c_{ij} & \cdots & c_{in} \\ \vdots & & \vdots & & \vdots \\ c_{m1} & \cdots & c_{mj} & \cdots & c_{mn} \end{bmatrix}$ then

a(B+C) $$= a\left(\begin{bmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ b_{i1} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & & \vdots & & \vdots \\ b_{m1} & \cdots & b_{mj} & \cdots & b_{mn} \end{bmatrix} + \begin{bmatrix} c_{11} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & & \vdots & & \vdots \\ c_{i1} & \cdots & c_{ij} & \cdots & c_{in} \\ \vdots & & \vdots & & \vdots \\ c_{m1} & \cdots & c_{mj} & \cdots & c_{mn} \end{bmatrix}\right)$$

$$= a\begin{bmatrix} b_{11}+c_{11} & \cdots & b_{1j}+c_{1j} & \cdots & b_{1n}+c_{1n} \\ \vdots & & \vdots & & \vdots \\ b_{i1}+c_{i1} & \cdots & b_{ij}+c_{ij} & \cdots & b_{in}+c_{in} \\ \vdots & & \vdots & & \vdots \\ b_{m1}+c_{m1} & \cdots & b_{mj}+c_{mj} & \cdots & b_{mn}+c_{mn} \end{bmatrix}$$

$$= \begin{bmatrix} a(b_{11}+c_{11}) & \cdots & a(b_{1j}+c_{1j}) & \cdots & a(b_{1n}+c_{1n}) \\ \vdots & & \vdots & & \vdots \\ a(b_{i1}+c_{i1}) & \cdots & a(b_{ij}+c_{ij}) & \cdots & a(b_{in}+c_{in}) \\ \vdots & & \vdots & & \vdots \\ a(b_{m1}+c_{m1}) & \cdots & a(b_{mj}+c_{mj}) & \cdots & a(b_{mn}+c_{mn}) \end{bmatrix}$$

$$= \begin{bmatrix} ab_{11}+ac_{11} & \cdots & ab_{1j}+ac_{1j} & \cdots & ab_{1n}+ac_{1n} \\ \vdots & & \vdots & & \vdots \\ ab_{i1}+ac_{i1} & \cdots & ab_{ij}+ac_{ij} & \cdots & ab_{in}+ac_{in} \\ \vdots & & \vdots & & \vdots \\ ab_{m1}+ac_{m1} & \cdots & ab_{mj}+ac_{mj} & \cdots & ab_{mn}+ac_{mn} \end{bmatrix}$$

$$= \begin{bmatrix} ab_{11} & \cdots & ab_{1j} & \cdots & ab_{1n} \\ \vdots & & \vdots & & \vdots \\ ab_{i1} & \cdots & ab_{ij} & \cdots & ab_{in} \\ \vdots & & \vdots & & \vdots \\ ab_{m1} & \cdots & ab_{mj} & \cdots & ab_{mn} \end{bmatrix} + \begin{bmatrix} ac_{11} & \cdots & ac_{1j} & \cdots & ac_{1n} \\ \vdots & & \vdots & & \vdots \\ ac_{i1} & \cdots & ac_{ij} & \cdots & ac_{in} \\ \vdots & & \vdots & & \vdots \\ ac_{m1} & \cdots & ac_{mj} & \cdots & ac_{mn} \end{bmatrix}$$

$$= a\begin{bmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ b_{i1} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & & \vdots & & \vdots \\ b_{m1} & \cdots & b_{mj} & \cdots & b_{mn} \end{bmatrix} + a\begin{bmatrix} c_{11} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & & \vdots & & \vdots \\ c_{i1} & \cdots & c_{ij} & \cdots & c_{in} \\ \vdots & & \vdots & & \vdots \\ c_{m1} & \cdots & c_{mj} & \cdots & c_{mn} \end{bmatrix} = aB+aC$$

Using more compact notation, just follow the ij^{th} entry:

$$[a(B+C)]_{ij} = a(b_{ij} + c_{ij}) = ab_{ij} + ac_{ij} = [aB]_{ij} + [aC]_{ij} = [aB + aC]_{ij}$$

OR

$$[a(B+C)]_{ij} = a([B]_{ij} + [C]_{ij}) = a[B]_{ij} + a[C]_{ij} = [aB]_{ij} + [aC]_{ij} = [aB + aC]_{ij}$$