Finding the Values of the Trigonometric Functions When One is Known

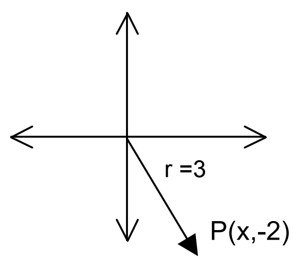
("Three Way Problem")

PROBLEM: Given $\sin\theta = -2/3$ and θ in Quadrant IV, find $\cos\theta$ and $\tan\theta$.

Method 1: Using the definition that uses a point on the terminal side.. Draw an angle in quadrant IV and assign values to x, y, r based on the value of the given trig. function. Find the missing unknown (x, y, or r) using the relationship $x^2 + y^2 = r^2$. Use the definitions to obtain the other trig. functions

Since $\sin\theta = \frac{-2}{3} = \frac{y}{r}$, take y = -2 and r = 3. Since $x^2 + y^2 = r^2$, $x^2 + (-2)^2 = 3^2$ yields $x = \pm\sqrt{5}$.

Since in Quadrant IV x>0, we get $x = \sqrt{5}$. Now $\cos\theta = \frac{x}{r} = \frac{\sqrt{5}}{3}$ and $\tan\theta = \frac{y}{x} = \frac{-2}{\sqrt{5}}$



Method 2. Using identites.

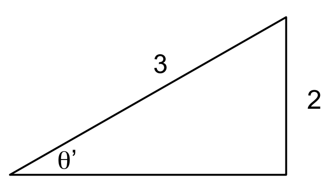
Since we know $\sin\theta = \frac{-2}{3}$, using the identity $\sin^2\theta + \cos^2\theta = 1$ we get $(-2/3)^2 + \cos^2\theta = 1$.

So $\cos^2\theta = 1 - 4/9 = 5/9$. Thus $\cos\theta = \pm\sqrt{\frac{5}{9}} = \pm\frac{\sqrt{5}}{3}$. But since we know $\cos\theta > 0$ in Quadrant IV, we get

$\cos\theta = \frac{\sqrt{5}}{3}$. Now $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-2/3}{\sqrt{5}/3} = \frac{-2}{\sqrt{5}}$.

Method 3. Using right triangles and reference angles. Draw a right triangle having θ's reference angle, θ' ,as an acute angle. Since $\sin\theta' = |\sin\theta|$ we can draw 2 of the three sides of the right triangle. Using the right triangle definitions, we can find the other trig. functions for the reference angle θ'. Now to get the values for θ we need only attach the appropriate sign based on the Quadrant of θ.

Since $\sin\theta = \frac{-2}{3}$, $\sin\theta' = \frac{2}{3} = \frac{opp}{hyp}$. Setting up a corresponding triangle we use Pythagorean theorem to get the side adjacent to θ' to be $\sqrt{5}$. Using the right triangle, $\cos\theta' = \frac{\sqrt{5}}{3}$ and $\tan\theta' = \frac{2}{\sqrt{5}}$. In Quadrant IV, cosine>0 and tangent <0 so $\cos\theta = \frac{\sqrt{5}}{3}$ and $\tan\theta = -\frac{2}{\sqrt{5}}$.



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