

Be sure to show all work neatly and follow instructions carefully; *no credit given if requested method is not used*. Clearly mark all answers. Express answers as ordered pairs/triples where appropriate. NO GRAPHING CALCULATORS, SCRATCH PAPER, BOOKS, NOTES, ELECTRONIC DICTIONARIES ETC.

(1) Given the following matrices:

$$A = \begin{bmatrix} 3 & 1 \\ 7 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 & 2 \\ -1 & 1 & 5 \\ -4 & -3 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 4 & -6 \\ 2 & 5 \end{bmatrix} \quad D = \begin{bmatrix} -5 & 0 & -2 \\ 3 & 1 & -7 \end{bmatrix} \quad E = \begin{bmatrix} -1 & -2 & 0 & 4 \\ 4 & 2 & 3 & 0 \\ 7 & -1 & -1 & 1 \\ 6 & 0 & 0 & 0 \end{bmatrix}$$

Find the following, if possible. (If not possible, say so.)

- | | |
|-------------|-------------|
| (a) DA | (b) A + C |
| (c) A C | (d) DB |
| (g) det (B) | (h) det (E) |

(2) Use Cramer's Rule to solve the following system. $\begin{cases} 3x - 3y = 5 \\ -x + 5y = 7 \end{cases}$

(3) Find the inverse of the matrix A.

$$A = \begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix}$$

Use A^{-1} to solve the system $\begin{cases} x - 2y - 4z = 2 \\ 2x - 3y - 6z = 0 \\ -3x + 6y + 15z = 1 \end{cases}$

(4) Solve: $\begin{cases} x^2 + 2y^2 - 7y = 0 \\ x^2 + y^2 = 10 \end{cases}$

(5) Solve using any of the methods discussed in class.

$$\begin{aligned} x + y - 10z &= -4 \\ -3x - 5y + 36z &= 10 \\ -x &+ 7z = 5 \end{aligned}$$

(6) Find the partial fraction decomposition of $\frac{2x - 4}{x(x - 1)^2}$

(7) Use matrix methods (Gaussian elimination or Gauss Jordan) to solve:

$$\begin{aligned} 3x + 2y - 5z &= 1 \\ 2x - 3y - 8z &= 1 \\ x + 5y + 2z &= 1 \end{aligned}$$

You must obtain row echelon form or reduced row echelon form. Be sure to label operations performed at each step.

Answers:

(1) (a) not possible (b) $\begin{bmatrix} 7 & -5 \\ 9 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 14 & -13 \\ 24 & -52 \end{bmatrix}$ (d) $\begin{bmatrix} -7 & 1 & -16 \\ 36 & 25 & -10 \end{bmatrix}$

(g) 51 (h) $-6 \begin{vmatrix} -2 & 0 & 4 \\ 2 & 3 & 0 \\ -1 & -1 & 1 \end{vmatrix} = 12$

(2) $D=12, Dx=46, Dy=26 \Rightarrow \left(\frac{23}{6}, \frac{13}{6} \right)$

(3) $A^{-1} = \begin{bmatrix} -3 & 2 & 0 \\ -4 & 1 & -\frac{2}{3} \\ 1 & 0 & \frac{1}{3} \end{bmatrix}, \vec{x} = A^{-1}\vec{b} = \begin{bmatrix} -6 \\ -\frac{26}{3} \\ \frac{7}{3} \end{bmatrix}$ so solution is $\left(-6, -\frac{26}{3}, \frac{7}{3} \right)$

(4) $(\pm\sqrt{6}, 2)$ (5) Dependent, answer not unique..(7t-5, 3t+1, t)

(6) $\frac{-4}{x} + \frac{4}{x-1} - \frac{2}{(x-1)^2}$ (7) (-2, 1, -1)