# Gaussian Elimination

Solve:

$$\begin{cases} x+2y-2z=-2 \quad (1) \\ -5x-9y+4z=3 \quad (2) \\ 3x+4y-5z=-3 \quad (3) \end{cases}$$

Eliminate x

$$\begin{cases} 5Eq(1) \rightarrow 5x+10y-10z=-10 \\ Eq.(2) \rightarrow -5x-9y+4z=3 \end{cases} \rightarrow ADD \rightarrow \quad y-6z=-7 \quad (4)$$

$$\begin{cases} -3Eq(1) \rightarrow -3x-6y+6z=6 \\ Eq.(3) \rightarrow \quad 3x+4y-5z=-3 \end{cases} \rightarrow ADD \rightarrow \quad -2y+z=3 \quad (5)$$

Elim. Y:

$$\begin{cases} 2Eq(4) \rightarrow 2y-12z=-14 \\ Eq.(5) \rightarrow \quad -2y+z=3 \end{cases} \quad ADD \rightarrow -11z=-11 \quad so \quad z=1$$

Augmented matrix

$$\begin{bmatrix} 1 & 2 & -2 & -2 \\ -5 & -9 & 4 & 3 \\ 3 & 4 & -5 & -3 \end{bmatrix}$$

Row operations

$$\cdots \begin{cases} 5R_1 + R_2 \rightarrow R_2 \\ -3R_1 + R_3 \rightarrow R_3 \end{cases} \Rightarrow \begin{bmatrix} 1 & 2 & -2 & -2 \\ 0 & 1 & -6 & -7 \\ 0 & -2 & 1 & 3 \end{bmatrix} \cdots \{ 2R_2 + R_3 \rightarrow R_3 \Rightarrow \begin{bmatrix} 1 & 2 & -2 & -2 \\ 0 & 1 & -6 & 7 \\ 0 & 0 & -11 & -11 \end{bmatrix} \cdots \{ \tfrac{1}{11} R_3 \rightarrow R_3 \Rightarrow$$

Row Echelon Form

$$\begin{bmatrix} 1 & 2 & -2 & -2 \\ 0 & 1 & -6 & 7 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Now write the corresponding system and use back substitution to solve.:

Example. (This is example 2 pg 664 done differently)

$$\begin{cases} 3x-y+5z=14 \\ x+2y-2z=10 \\ x-y+3z=4 \end{cases} \qquad \begin{bmatrix} 3 & -1 & 5 & 14 \\ 1 & 2 & -2 & 10 \\ 1 & -1 & 3 & 4 \end{bmatrix} \quad R_1 \leftrightarrow R_3 \quad \begin{bmatrix} 1 & -1 & 3 & 4 \\ 1 & 2 & -2 & 10 \\ 3 & -1 & 5 & 14 \end{bmatrix}$$

Get a "1" in the upper left  Use that "1" to get "0" below