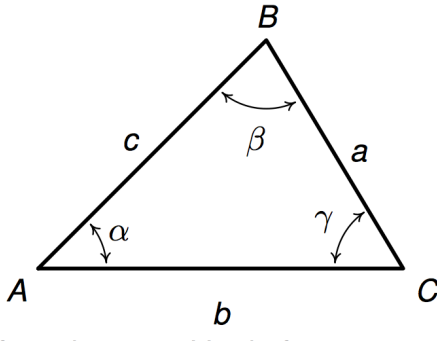


## UNIT 4 – LAW OF SINES AND COSINES, VECTORS, POLAR GRAPHS

UNIT 4 - HW Checklist		
NOTES	completed and in order given including completed worksheets:	10
	WS page 13-15: Law of Sines/Cosines Applications	5
	WS page25: Vector Resultant Problems	5
	Sample Test	10
11.3ii	41, 43, 46, 49, 51, 54, 57-71 odd, 87-101 odd, 131-145 odd, 155-161d	5
12.1	1, 5, 6, 7, 8, 10, 21, 23,24,26,32,33	5
12.2	1, 5, 6, 7, 14, 16, 23	5
12.3	1, 9,11,14,15,17,18,19,20, 25, 26, 32-34, 38, 40, 43, 44,47,49	5
12.3ii	53, 57, 60	5
12.4	1, 3, 9, 13, (dont need to find q) , 21, 23	5
13.1	1, 6, 10, 13, 17, 22, 27, 37-43, 57, 62, 64, 66, 77, 81, 85, 89	5
13.2	1-13 odd, 14 . (note: You won't be given the name of the graph on tes	5
		70

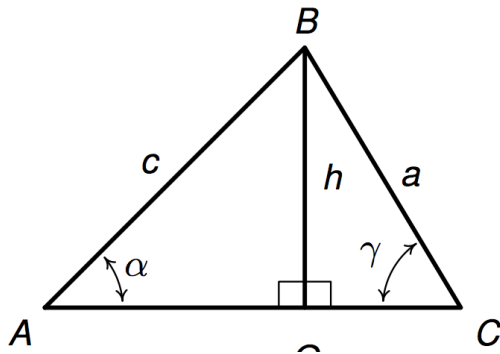
Unit 4– Law of Sines & Cosines, Vectors, Polar Graphs, Parametric Eqns
--

The next two sections discuss how we can “solve” (find missing parts) of \_\_\_\_\_ (non-right) triangles.



12.1 Law of Sines
-------------------

If we create right triangles by dropping a perpendicular from B to the side AC, we can use what we know about right triangles to find parts of triangle ABC.



(Book show additional cases/figures.)

**Theorem 12.1. The Law of Sines:** Given a triangle with angle-side opposite pairs  $(\alpha, a)$ ,  $(\beta, b)$  and  $(\gamma, c)$ , the following ratios hold:

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c} \quad \text{or, equivalently,} \quad \frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

**Law of Sines Examples**

(I will use the book's examples here because they are accompanied by useful pictures.)

**Example 12.1.2.** Solve the following triangles. Give exact answers and decimal approximations (rounded to hundredths) and sketch the triangle.

1)  $\alpha = 120^\circ$ ,  $a = 7$ ,  $\beta = 45^\circ$

2)  $\alpha = 85^\circ$ ,  $c = 5.25$ ,  $\beta = 30^\circ$  (see book pg 1026)

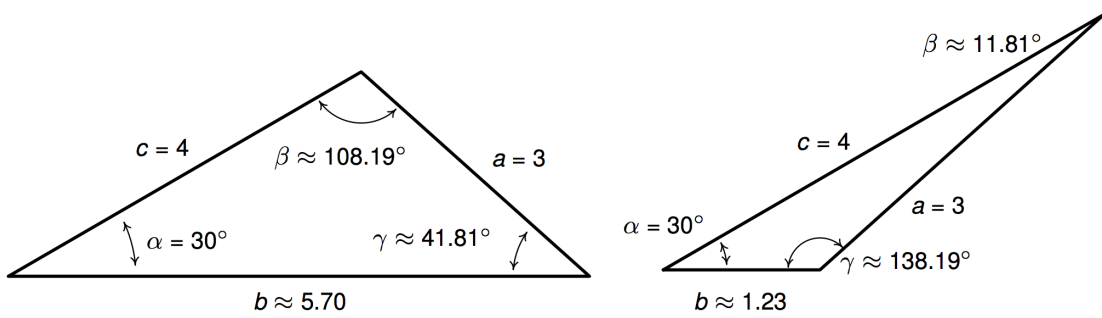
*See application problem that follows, then return to consider special cases that occur with Law of Sines*

3)  $\alpha = 30^\circ$ ,  $a = 1$ ,  $c = 4$

Law of Sines Examples cont'd

4) See in book (page 1027)

5)  $\alpha = 30^\circ$ ,  $a = 3$ ,  $c = 4$



6)  $\alpha = 30^\circ$ ,  $a = 4$ ,  $c = 4$

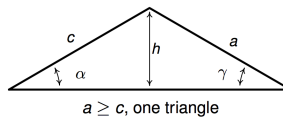
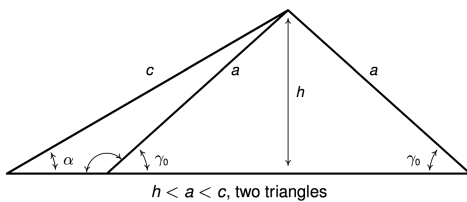
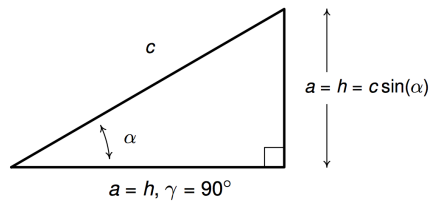
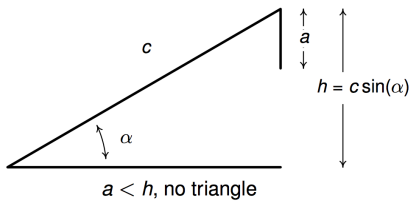
Examples 3-6 show all cases of the following special case.

**The Ambiguous Case SSA**

You do not need to memorize cases. You just need to be aware that when you solve the trig equation for an angle, there may be one solution, two solutions, or no solution.

**Theorem 12.2.** Suppose  $(\alpha, a)$  and  $(\gamma, c)$  are intended to be angle-side pairs in a triangle where  $\alpha$ ,  $a$  and  $c$  are given. Let  $h = c \sin(\alpha)$

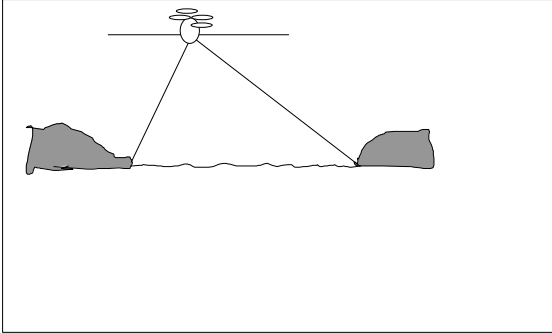
- If  $a < h$ , then no triangle exists which satisfies the given criteria.
- If  $a = h$ , then  $\gamma = 90^\circ$  so exactly one (right) triangle exists which satisfies the criteria.
- If  $h < a < c$ , then two distinct triangles exist which satisfy the given criteria.
- If  $a \geq c$ , then  $\gamma$  is acute and exactly one triangle exists which satisfies the given criteria



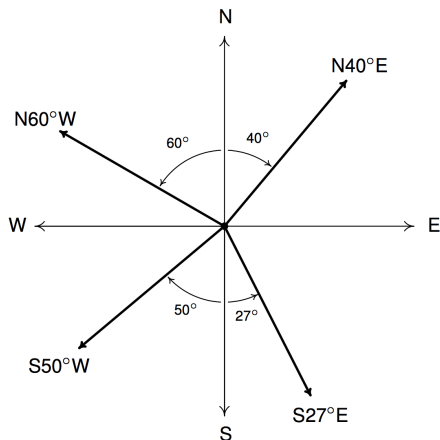
**Application Problems:**

Many problems we could do previously considering right triangles can now be done more directly using Law of Sines

A helicopter hovers 400 feet above a river. The angle of depression from the helicopter to the west bank is  $73^\circ$ , while the angle of depression from the helicopter to the east bank is  $44^\circ$ . (Remember, angle of depression is measured from the line of sight to the horizontal). Find the width of the river (exact and approximate.)



In physical problems, direction of travel is often given in bearings.



Example Bearings:

27. A hiker starts walking due west from Sasquatch Point and gets to the Chupacabra Trailhead before she realizes that she hasn't reset her pedometer. From the Chupacabra Trailhead she hikes for 5 miles along a bearing of  $N53^\circ W$  which brings her to the Muffin Ridge Observatory. From there, she knows a bearing of  $S65^\circ E$  will take her straight back to Sasquatch Point. How far will she have to walk to get from the Muffin Ridge Observatory to Sasquatch Point? What is the distance between Sasquatch Point and the Chupacabra Trailhead?

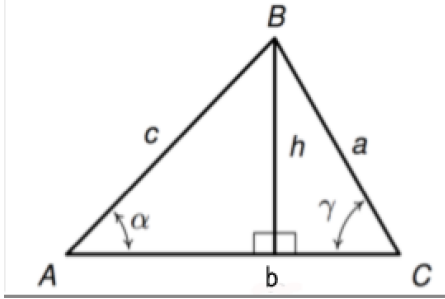
See book page 1032 for another example using bearings.

Example: Grade of hill (pg 1034)

22. Using a right triangle with a horizontal leg of length 100 and vertical leg with length 7, show that a 7% grade means that the road (hypotenuse) makes about a  $4^\circ$  angle with the horizontal. (It will not be exactly  $4^\circ$ , but it's pretty close.)

Application: Area of Triangle:
--------------------------------

Suppose we are asked to find the area of the following triangle.



Area of the triangle is  $A = \frac{1}{2}bh$ . But suppose we are given only  $b$ ,  $c$ ,  $\alpha$ . How would we find the area?

This leads to a new formula for finding area of a triangle when we know any two sides and the included angle (the angle between those two sides)

Example:



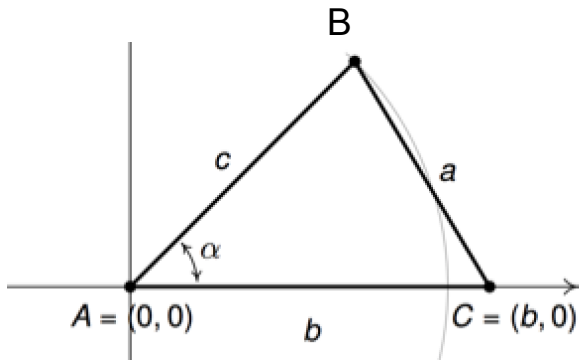
## 12.1 Law of Cosines

Example: Use the Law of Sines to find the remaining parts of the triangle given by  $a=4$ ,  $b=3$ ,  $c=6$

## Development of the Law of Cosines

Can we find a relationship relating the sides of an oblique triangle? Suppose we superimpose a coordinate system onto a general triangle as shown.

What are the coordinates of point B? \_\_\_\_\_



**Theorem 12.4. Law of Cosines:** Given a triangle with angle-side opposite pairs  $(\alpha, a)$ ,  $(\beta, b)$  and  $(\gamma, c)$ , the following equations hold

$$a^2 = b^2 + c^2 - 2bc \cos(\alpha) \quad b^2 = a^2 + c^2 - 2ac \cos(\beta) \quad c^2 = a^2 + b^2 - 2ab \cos(\gamma)$$

or, solving for the cosine in each equation, we have

$$\cos(\alpha) = \frac{b^2 + c^2 - a^2}{2bc} \quad \cos(\beta) = \frac{a^2 + c^2 - b^2}{2ac} \quad \cos(\gamma) = \frac{a^2 + b^2 - c^2}{2ab}$$

Note: you should not feel the need to memorize all 6 of these formulas! If you know the pattern of the first one, the rest follow.

Unit 4

Reconsider earlier Example: Find the remaining parts of the triangle given by  $a=4$ ,  $b=3$ ,  $c=6$

Tip: It is helpful to find the \_\_\_\_\_ first.  
(If we find the largest angle first, the others must be acute)

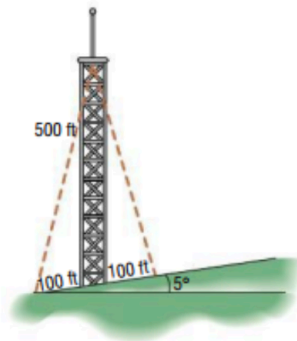
Example: Find remaining parts:  $a=2$ ,  $b=3$ ,  $\gamma=60^\circ$

Example pg 1046

24. The SS Bigfoot leaves a harbor bound for Nessie Island which is 300 miles away at a bearing of  $N32^\circ E$ . A storm moves in and after 100 miles, the captain of the Bigfoot finds he has drifted off course. If his bearing to the harbor is now  $S70^\circ W$ , how far is the SS Bigfoot from Nessie Island? Round your answer to the nearest hundredth of a mile. What course should the captain set to head to the island? Round your angle to the nearest tenth of a degree.

---

**Finding the Length of a Guy Wire** A radio tower 500 feet high is located on the side of a hill with an inclination to the horizontal of  $5^\circ$ . See the figure. How long should two guy wires be if they are to connect to the top of the tower and be secured at two points 100 feet directly above and directly below the base of the tower?



Law of Sines and Cosines Worksheet
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*(note: answers have not been checked)*

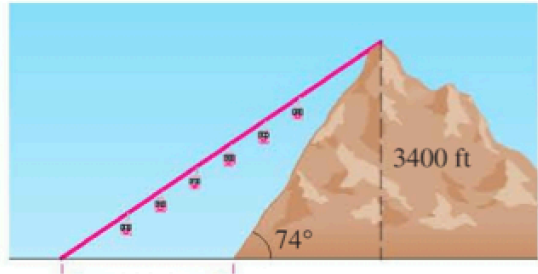
- (1) Two cars leave a city at the same time and travel along straight highways that differ in direction by  $128^\circ$ . If their speeds are 60mph and 50mph respectively, how far apart are the cars at the end of 20 minutes? (Exact and approximate.) *Approx ans: 33 miles*

- (2) A man looks up and sees an airplane flying in his direction at a level altitude of 2 miles. He watches the airplane for a few minutes. During that period of time he notices that the angle of elevation to the airplane changes from  $45^\circ$  to  $60^\circ$ . How far has the plane traveled in that time?

$$\text{Ans: } \frac{8 \sin 15^\circ}{\sqrt{6}} \approx 0.845 \text{ miles}$$

(3)

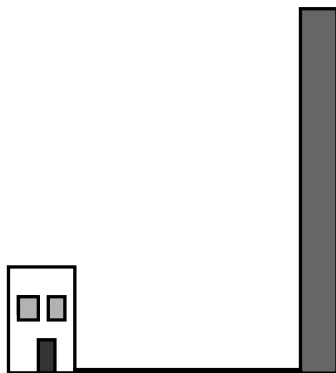
**Cable Car** A steep mountain is inclined  $74^\circ$  to the horizontal and rises 3400 ft above the surrounding plain. A cable car is to be installed from a point 800 ft from the base to the top of the mountain, as shown. Find the shortest length of cable needed.



*approx ans: 3835 ft.*

(4) If a tower is viewed from the top of a 50 foot building, the angle of elevation to the top of the tower is  $59^\circ$ . If viewed from the ground floor of the same building, the angle of elevation to the top of the tower is  $62^\circ$ . Find the height of the tower. (Show an exact answer and an approximate.)

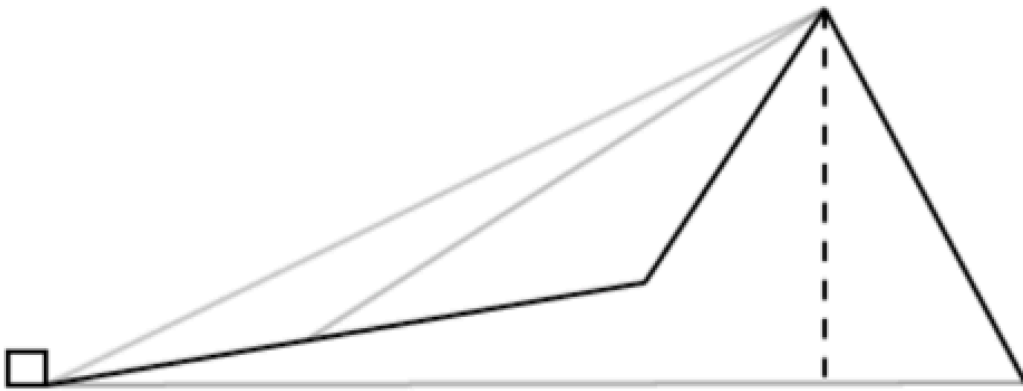
$$\text{ans: } \frac{50 \sin 149^\circ}{\sin 3^\circ} \sin 62^\circ \approx 434.5 \text{ ft}$$



(5)

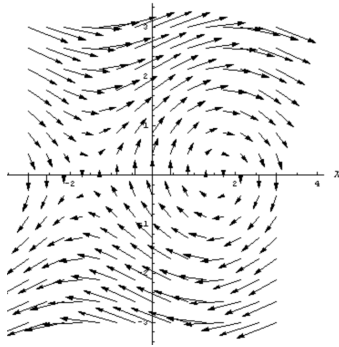
A man observes that the angle of elevation of a mountain peak from his house is  $26^\circ$ . Leaving the house he walks 2000 ft. up a slope of  $10^\circ$  directly towards the mountain and then finds that the angle of elevation of the peak to be  $31^\circ$ . What is the height of the mountain peak (relative to the house). Exact and approximate.

$$\text{ans: } \frac{2000 \sin 59^\circ}{\sin 5^\circ} \sin 26^\circ \approx 3605 \text{ ft}$$



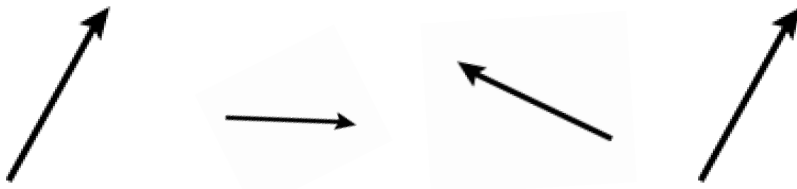
12.3 i Vectors

A vector is a mathematical object that allows us to represent both \_\_\_\_\_ and \_\_\_\_\_ . Vectors are often used physics and engineering.

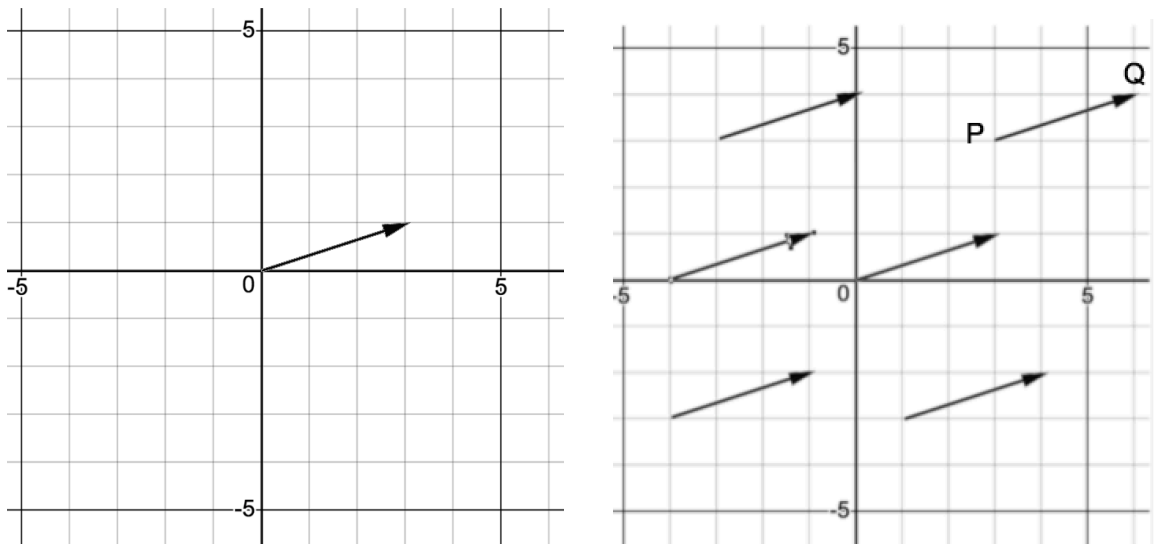


Geometric Representation of a vector:

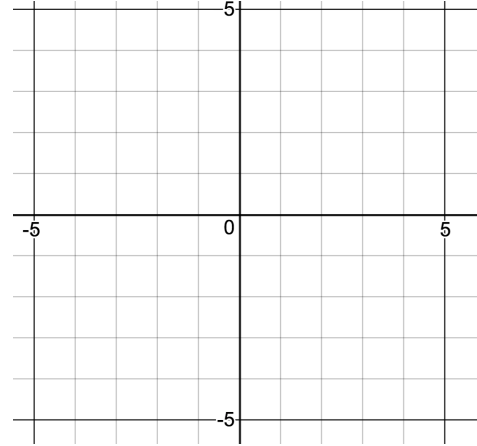
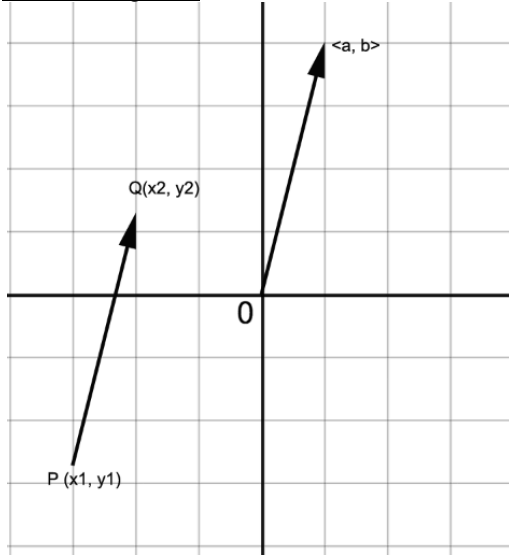
Initial point, terminal point, notation, magnitude, direction, equal vectors



Algebraic (Component) Representation of a Vector: Superimpose a coordinate system

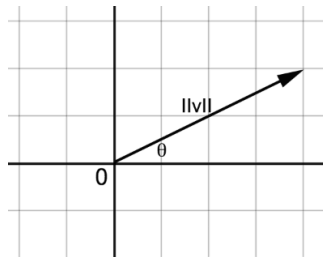


Computing the components of a vector  $\vec{v}$  with representative  $\overline{PQ}$  where the coordinates of points P and Q are given.

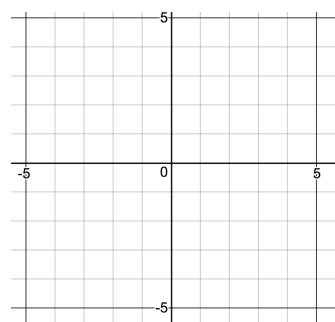
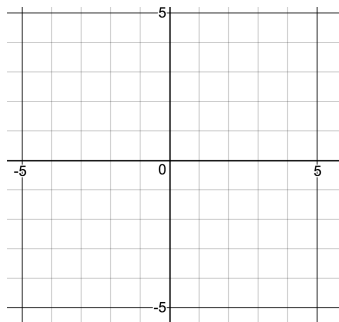


Converting: Magnitude/Direction to Component Form (if  $\theta$  in standard position)

Given  $\|\vec{v}\|$ ,  $\theta$  find \_\_\_\_\_



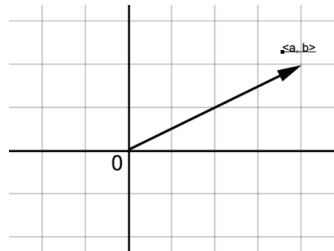
Examples:





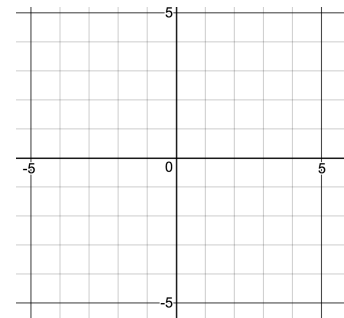
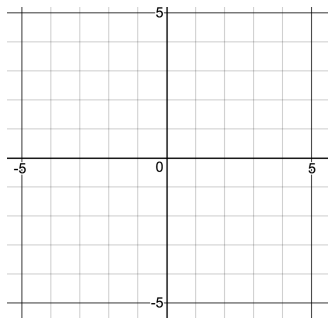
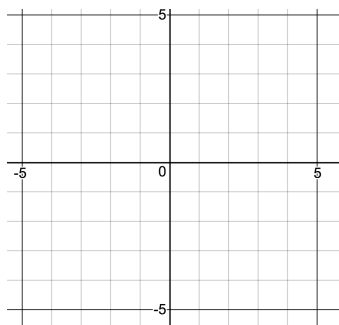
Converting: Component Form to Magnitude/Direction Form

Given  $\vec{v} = \langle a \ b \rangle$  find \_\_\_\_\_



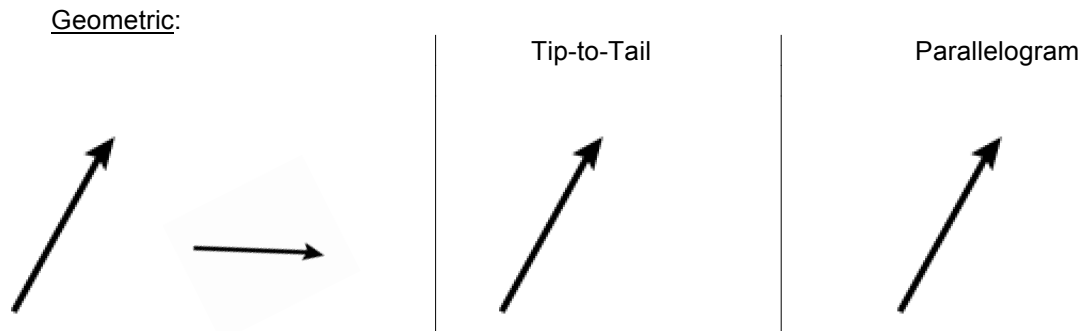
\*\*\* What quadrant is  $\theta$  in?

Examples:



Vector Operations

Addition Given  $\vec{v}$ ,  $\vec{w}$ , how do we define  $\vec{v} + \vec{w}$ ?

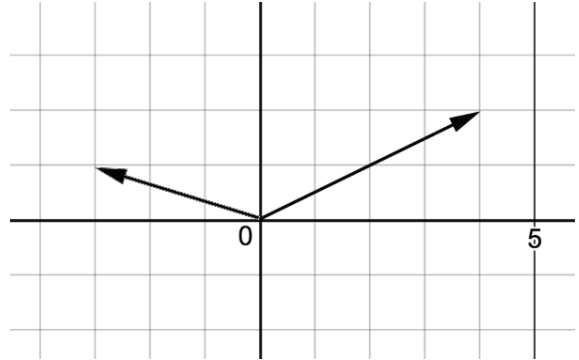


Addition (cont'd)Algebraic Definition:

**Definition 12.2.** Suppose  $\vec{v} = \langle v_1, v_2 \rangle$  and  $\vec{w} = \langle w_1, w_2 \rangle$ . The vector  $\vec{v} + \vec{w}$  is defined by

$$\vec{v} + \vec{w} = \langle v_1 + w_1, v_2 + w_2 \rangle$$

Example:

**Theorem 12.6. Properties of Vector Addition**

- **Commutative Property:** For all vectors  $\vec{v}$  and  $\vec{w}$ ,  $\vec{v} + \vec{w} = \vec{w} + \vec{v}$ .
- **Associative Property:** For all vectors  $\vec{u}$ ,  $\vec{v}$  and  $\vec{w}$ ,  $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ .
- **Identity Property:** For all vectors  $\vec{v}$ ,

$$\vec{v} + \vec{0} = \vec{0} + \vec{v} = \vec{v}.$$

The vector  $\vec{0}$  acts as the additive identity for vector addition.

- **Inverse Property:** For every vector  $\vec{v} = \langle v_1, v_2 \rangle$ , the vector  $\vec{w} = \langle -v_1, -v_2 \rangle$  satisfies

$$\vec{v} + \vec{w} = \vec{w} + \vec{v} = \vec{0}.$$

That is, the additive inverse of a vector is the vector of the additive inverses of its components.

Zero Vector: \_\_\_\_\_

Proving Vector Properties:

Scalar Multiplication

Geometric:



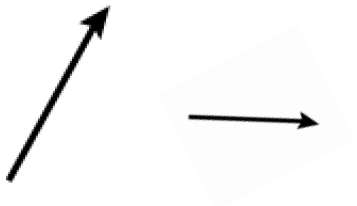
Algebraic

**Definition 12.3.** If  $k$  is a real number and  $\vec{v} = \langle v_1, v_2 \rangle$ , we define  $k\vec{v}$  by

$$k\vec{v} = k \langle v_1, v_2 \rangle = \langle kv_1, kv_2 \rangle$$

Subtraction

Geometric:



Adding Opposite



Parallelogram



Algebraic:

**Theorem 12.7. Properties of Scalar Multiplication**

- **Associative Property:** For every vector  $\vec{v}$  and scalars  $k$  and  $r$ ,  $(kr)\vec{v} = k(r\vec{v})$ .
- **Identity Property:** For all vectors  $\vec{v}$ ,  $1\vec{v} = \vec{v}$ .
- **Additive Inverse Property:** For all vectors  $\vec{v}$ ,  $-\vec{v} = (-1)\vec{v}$ .
- **Distributive Property of Scalar Multiplication over Scalar Addition:**  
For every vector  $\vec{v}$  and scalars  $k$  and  $r$ ,

$$(k + r)\vec{v} = k\vec{v} + r\vec{v}$$

- **Distributive Property of Scalar Multiplication over Vector Addition:**  
For all vectors  $\vec{v}$  and  $\vec{w}$  and scalars  $k$ ,

$$k(\vec{v} + \vec{w}) = k\vec{v} + k\vec{w}$$

- **Zero Product Property:** If  $\vec{v}$  is vector and  $k$  is a scalar, then

$$k\vec{v} = \vec{0} \quad \text{if and only if} \quad k = 0 \quad \text{or} \quad \vec{v} = \vec{0}$$

See example proof page 1055.

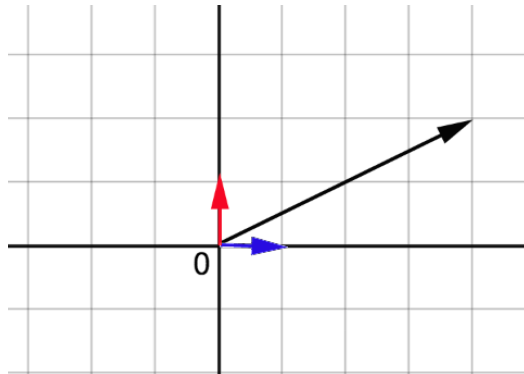
Unit Vector

A vector  $\vec{v}$  is called a unit vector if  $\|\vec{v}\| =$  \_\_\_\_\_

Standard basis unit vectors:  $\vec{i} = \hat{i} =$  \_\_\_\_\_

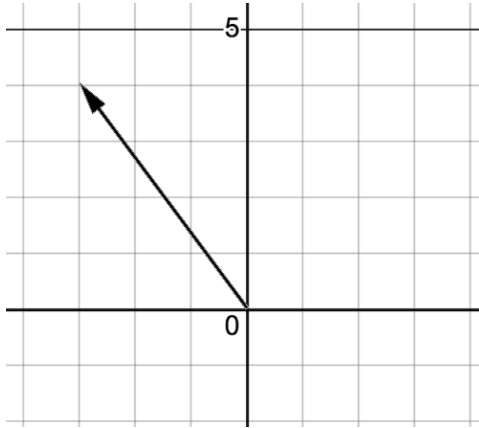
$\vec{j} = \hat{j} =$  \_\_\_\_\_

All vectors  $\vec{v} = \langle a \ b \rangle$  can be written in the form  $\vec{v} = a\vec{i} + b\vec{j}$



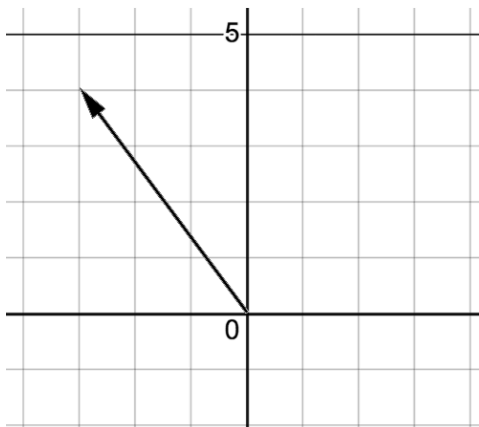
We are often interested to find unit vectors in a specified direction.

EX: Find a unit vector in the direction of  $\vec{v} = \langle -3, 4 \rangle$



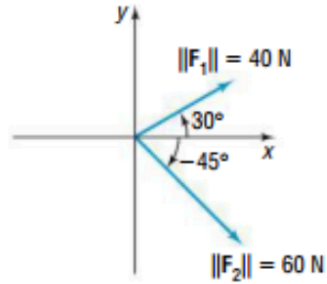
In general, a unit vector in the direction of  $\vec{v}$  is given by \_\_\_\_\_

EX: Find a vector of length 7 in the direction of  $\vec{v} = \langle -3, 4 \rangle$



## 12.3ii Applications – Resultants of Forces

- 74. Force Vectors** A man pushes a wheelbarrow up an incline of  $20^\circ$  with a force of 100 pounds. Express the force vector  $\mathbf{F}$  in terms of  $\mathbf{i}$  and  $\mathbf{j}$ .
- 75. Resultant Force** Two forces of magnitude 40 newtons (N) and 60 N act on an object at angles of  $30^\circ$  and  $-45^\circ$  with the positive  $x$ -axis, as shown in the figure. Find the direction and magnitude of the resultant force; that is, find  $\mathbf{F}_1 + \mathbf{F}_2$ .



---

**Example 12.3.5.** A plane leaves an airport with an airspeed of 175 miles per hour with bearing  $N40^\circ E$ . A 35 mile per hour wind is blowing at a bearing of  $S60^\circ E$ . Find the true speed of the plane, rounded to the nearest mile per hour, and the true bearing of the plane, rounded to the nearest degree.

(page1059)

**Example 12.3.6.** A 50 pound speaker is suspended from the ceiling by two support braces. If one of them makes a  $60^\circ$  angle with the ceiling and the other makes a  $30^\circ$  angle with the ceiling, what are the tensions on each of the supports?

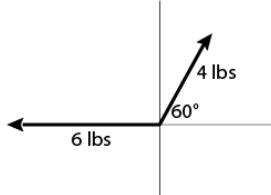
(page 1062)

(note: answers have not been checked)

Worksheet: Additional Vector Resultant Practice.

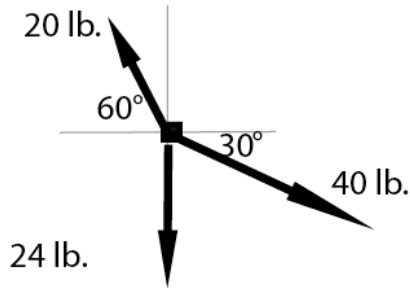
- (1) Two forces act on an object as shown. Sketch the resultant. Find the magnitude and the direction of the resultant.(exact and approx.)

$$\text{Ans: } 2\sqrt{7} \text{ lbs, } \theta = 180^\circ + \tan^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$



- (2) Three forces act on an object as shown. Find the magnitude and the direction of the resultant.

$$\text{Ans: } \|\vec{R}\| \approx 36.32 \text{ lbs, } \theta \approx -47.3^\circ$$



- (3) An airplane is traveling at a constant airspeed of 500 mph in the direction N45°W. If wind is blowing directly northward at a rate of 40 mph, what is the actual speed and direction of the airplane?

$$\text{Ans: Approx } 190.4 \text{ mph, } N42.47^\circ W$$



## 12.4 The Dot Product (Scalar Product, Inner Product)

**Definition 12.7.** Given vectors  $\vec{v} = \langle v_1, v_2 \rangle$  and  $\vec{w} = \langle w_1, w_2 \rangle$ , the **dot product** of  $\vec{v}$  and  $\vec{w}$  is given by

$$\vec{v} \cdot \vec{w} = \langle v_1, v_2 \rangle \cdot \langle w_1, w_2 \rangle = v_1 w_1 + v_2 w_2$$

Examples:

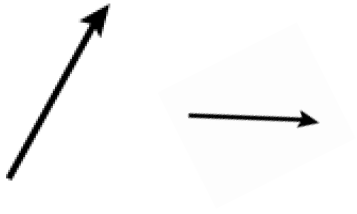
### Theorem 12.10. Properties of the Dot Product

- **Commutative Property:** For all vectors  $\vec{v}$  and  $\vec{w}$ ,  $\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$ .
- **Distributive Property:** For all vectors  $\vec{u}$ ,  $\vec{v}$  and  $\vec{w}$ ,  $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$ .
- **Scalar Property:** For all vectors  $\vec{v}$  and  $\vec{w}$  and scalars  $k$ ,  $(k\vec{v}) \cdot \vec{w} = k(\vec{v} \cdot \vec{w}) = \vec{v} \cdot (k\vec{w})$ .
- **Relation to Magnitude:** For all vectors  $\vec{v}$ ,  $\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$ .

Example Proof

Applications of Dot Products.

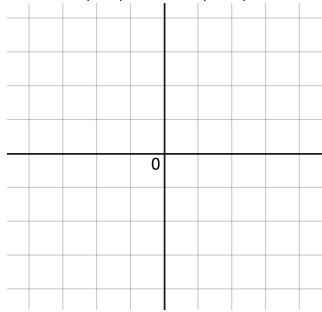
Seemingly unrelated problem: Find “the angle” between two vectors



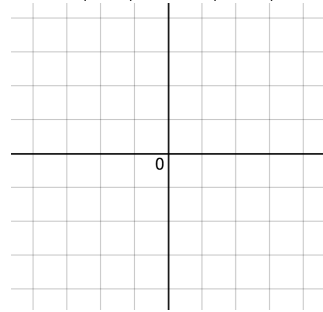
$$\cos(\theta) = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|} \quad \vec{v} \cdot \vec{w} = \|\vec{v}\| \|\vec{w}\| \cos(\theta) \quad \vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2$$

Example: Find the angle between

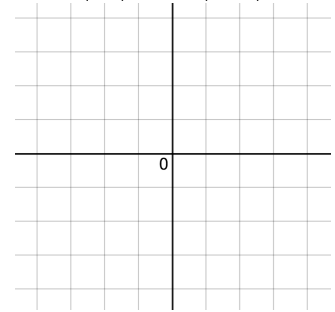
a)  $\vec{v} = \langle 3, 1 \rangle$     $\vec{w} = \langle 2, 4 \rangle$



b)  $\vec{v} = \langle -2, 1 \rangle$     $\vec{w} = \langle 3, -4 \rangle$



c)  $\vec{v} = \langle 2, 3 \rangle$     $\vec{w} = \langle -3, 2 \rangle$



$$\vec{v} \cdot \vec{w} > 0$$

$$\underline{\hspace{2cm}} < \theta < \underline{\hspace{2cm}}$$

$$\vec{v} \cdot \vec{w} < 0$$

$$\underline{\hspace{2cm}} < \theta < \underline{\hspace{2cm}}$$

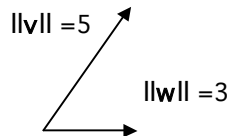
$$\vec{v} \cdot \vec{w} = 0$$

$$\theta = \underline{\hspace{2cm}}$$

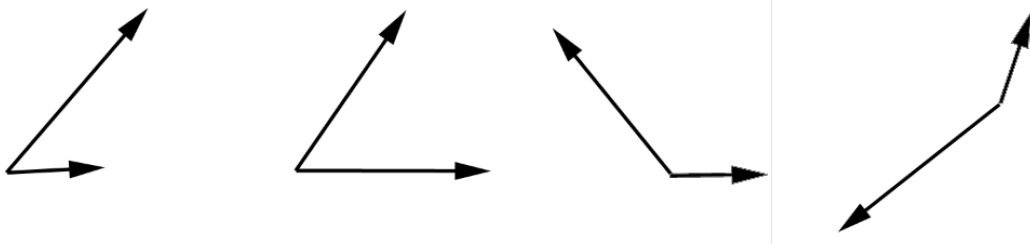
Another way to find the dot product, depending on what info you are given.

Example: Find  $\vec{v} \cdot \vec{w}$

given the vectors  $\vec{v}$  and  $\vec{w}$  as shown, with the angle between  $\vec{v}$  and  $\vec{w}$  equals  $60^\circ$ ,



Orthogonal Projections



Derivation of the formula for finding the projection of  $\vec{v}$  onto  $\vec{w}$ ,  $proj_{\vec{w}}(\vec{v})$

First notice that  $proj_{\vec{w}}(\vec{v})$  is either in the direction of  $\vec{w}$  or in the opposite direction, thus



$proj_{\vec{w}}(\vec{v}) =$  \_\_\_\_\_ or  $proj_{\vec{w}}(\vec{v}) =$  \_\_\_\_\_

Suppose we knew the length of the projection, call it L. ( $L = \|proj_{\vec{w}}(\vec{v})\|$ ). Then similar to the example on page 22,

$proj_{\vec{w}}(\vec{v}) =$  \_\_\_\_\_ or  $proj_{\vec{w}}(\vec{v}) =$  \_\_\_\_\_

Now let's find L



$\frac{L}{\|\vec{v}\|} =$  \_\_\_\_\_  $\frac{L}{\|\vec{v}\|} =$  \_\_\_\_\_

$L =$  \_\_\_\_\_  $L =$  \_\_\_\_\_

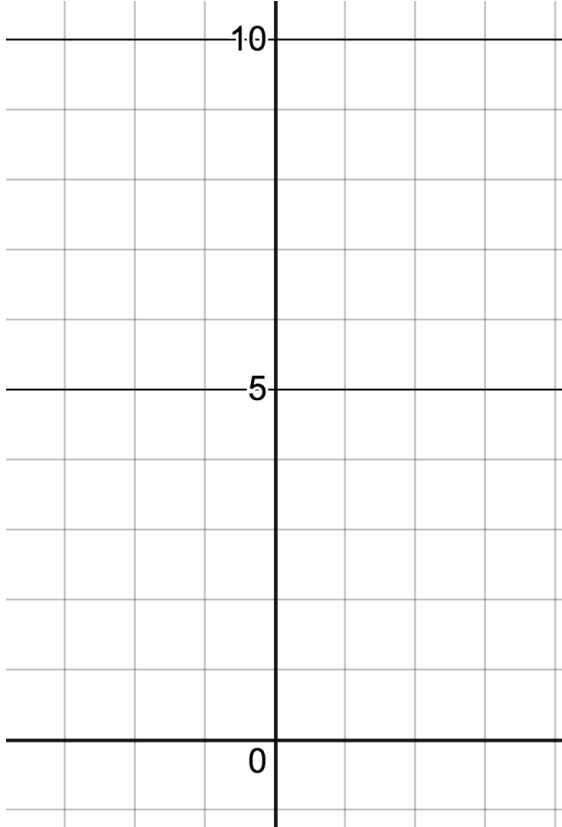
$L =$  \_\_\_\_\_ so as a dot product,  $L =$  \_\_\_\_\_

$proj_{\vec{w}}(\vec{v}) =$  \_\_\_\_\_ or  $proj_{\vec{w}}(\vec{v}) =$  \_\_\_\_\_

**Theorem 12.14. Alternate Formulas for Vector Projections:** If  $\vec{v}$  and  $\vec{w}$  are nonzero vectors then

$$\text{proj}_{\vec{w}}(\vec{v}) = (\vec{v} \cdot \hat{w})\hat{w} = \left( \frac{\vec{v} \cdot \vec{w}}{\|\vec{w}\|^2} \right) \vec{w} = \left( \frac{\vec{v} \cdot \vec{w}}{\vec{w} \cdot \vec{w}} \right) \vec{w}$$

**Example 12.4.4.** Let  $\vec{v} = \langle 1, 8 \rangle$  and  $\vec{w} = \langle -1, 2 \rangle$ . Find  $\vec{p} = \text{proj}_{\vec{w}}(\vec{v})$ . Check your answer geometrically.



Orthogonal Vectors/ Parallel Vectors

If  $\vec{v}$  and  $\vec{w}$  are orthogonal, then \_\_\_\_\_

If  $\vec{v}$  and  $\vec{w}$  are parallel, then \_\_\_\_\_

Ex: Are the vectors  $\vec{v} = \langle 4, -1 \rangle$  and  $\vec{w} = \langle -3, 2 \rangle$  orthogonal?

Ex: Are the vectors  $\vec{v} = \langle 7, -2 \rangle$  and  $\vec{w} = \langle 4, 14 \rangle$  orthogonal?

Ex: Find  $x$  such that  $\vec{v} = \langle 4, x \rangle$  and  $\vec{w} = \langle -5, 2 \rangle$  orthogonal.

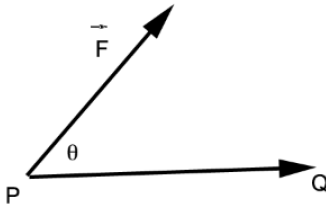
Ex: Are the vectors  $\vec{v} = \langle -12, 8 \rangle$  and  $\vec{w} = \langle -3, 2 \rangle$  parallel?

## Work

In physics, if a constant force  $F$  is exerted over a distance  $d$ , the work  $W$  done by the force is given by  $W=Fd$ . Here the assumption is that the force being applied is in the direction of motion.

22. Find the work done lifting a 10 pound book 3 feet straight up into the air. Assume the force of gravity is acting straight downwards.

If the force is applied not in the direction of motion, we need to determine how much of the force is in the direction of motion.



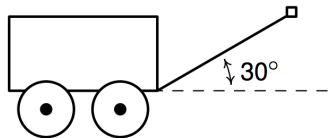
**Theorem 12.16. Work as a Dot Product:** Suppose a constant force  $\vec{F}$  is applied along the vector  $\vec{PQ}$ . The work  $W$  done by  $\vec{F}$  is given by

$$W = \vec{F} \cdot \vec{PQ} = \|\vec{F}\| \|\vec{PQ}\| \cos(\theta),$$

where  $\theta$  is the angle between  $\vec{F}$  and  $\vec{PQ}$ .

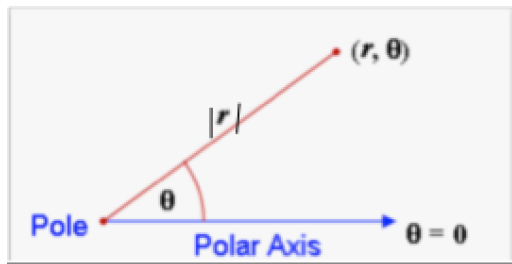
We test out our formula for work in the following example.

**Example 12.4.5.** Taylor exerts a force of 10 pounds to pull her wagon a distance of 50 feet over level ground. If the handle of the wagon makes a  $30^\circ$  angle with the horizontal, how much work did Taylor do pulling the wagon? Assume the force of 10 pounds is exerted at a  $30^\circ$  angle for the duration of the 50 feet.

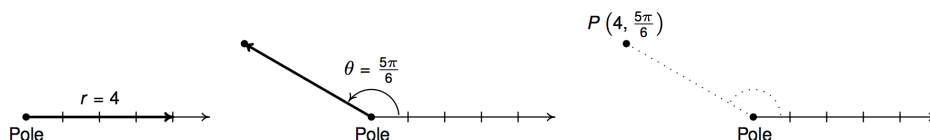


### 13.1 Polar Coordinates

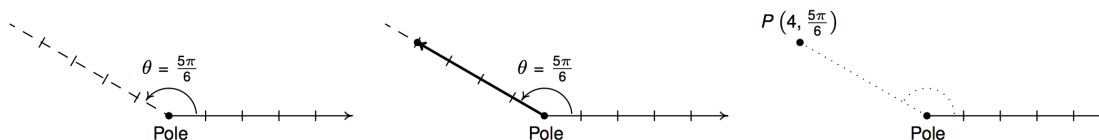
Up until now, we have used Cartesian or Rectangular Coordinates to locate a point in two dimensions. Here we will discuss a new way.



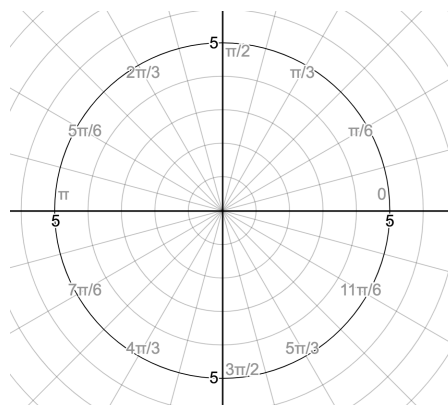
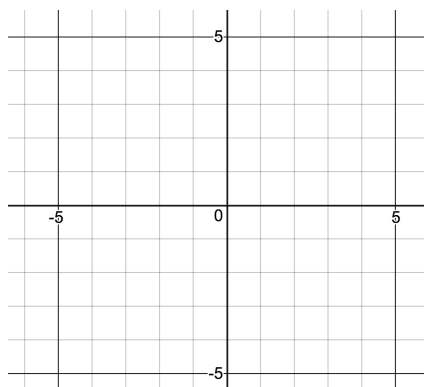
For example, if we wished to plot the point  $P$  with polar coordinates  $(4, \frac{5\pi}{6})$ , we'd start at the pole, move out along the polar axis 4 units, then rotate  $\frac{5\pi}{6}$  radians counter-clockwise.



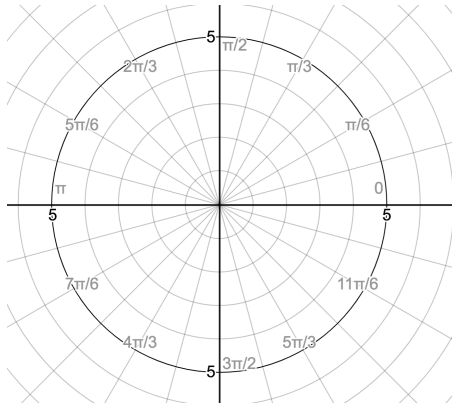
We may also visualize this process by thinking of the rotation first.<sup>3</sup> To plot  $P(4, \frac{5\pi}{6})$  this way, we rotate  $\frac{5\pi}{6}$  counter-clockwise from the polar axis, then move outwards from the pole 4 units. Essentially we are locating a point on the terminal side of  $\frac{5\pi}{6}$  which is 4 units away from the pole.



We can approximate such a point on rectangular graph paper or we can use polar graph paper.

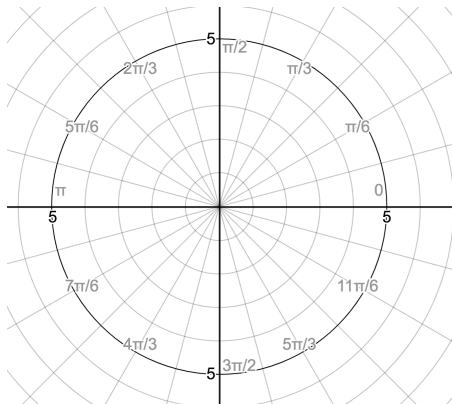


Example: Plot the polar points:



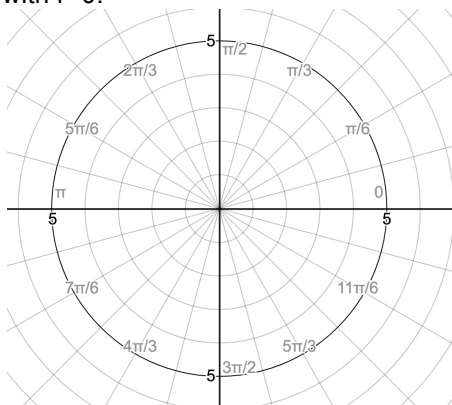
Note, representation of polar points is not unique.

What if  $r < 0$ ? Plot the polar points :



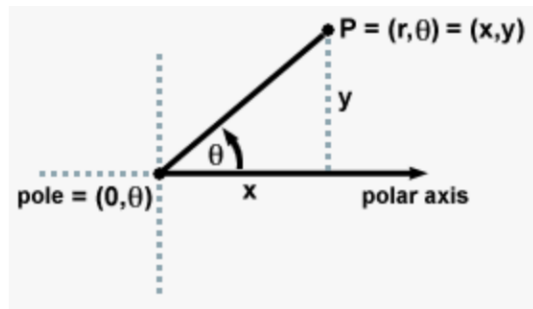
Often we are asked to find other representations of a given polar point.

Example: Find two other representations of the polar point \_\_\_\_\_, one with  $r > 0$  and one with  $r < 0$ .





## Converting Points



Polar to Rectangular

Rectangular to Polar



## 13.2 Graphing polar equations

The more we work with polar graphs, the more we will recognize the equation of basic polar graphs and be able graph them more efficiently. Here are some strategies we can use.

- 1) Do we recognize the equation? If so, use that knowledge to graph quickly.
- 2) Try converting the polar equation to rectangular to see if it is a known graph.
- 3) Use an auxiliary rectangular graph.
- 4) Last resort, plot points.

Example:  $r = 3$

$$r = a$$

Example:  $\theta = \frac{\pi}{6}$

$$\theta = k$$

Example:  $r = 4 \cos(\theta)$

OFFSET CIRCLE

$$r = 2a \cos(\theta)$$

$$r = 2a \sin(\theta)$$

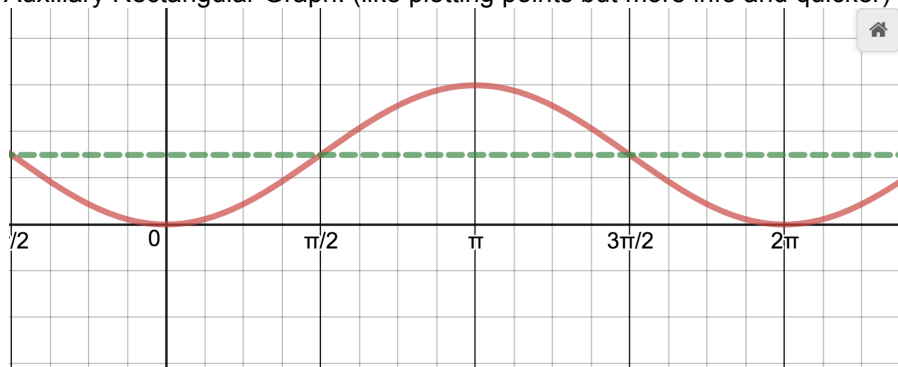
Ex: Graph  $r = 6 \sin(\theta)$  now that we recognize this type of equation.

Example:  $r = \frac{3}{2} - \frac{3}{2} \cos(\theta)$

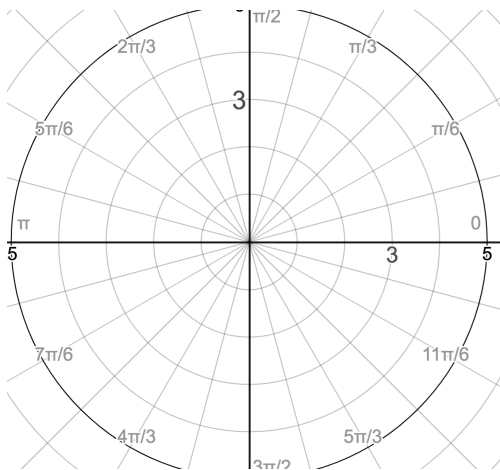
Convert:

Plot points:

Auxiliary Rectangular Graph: (like plotting points but more info and quicker)



Polar Graph:



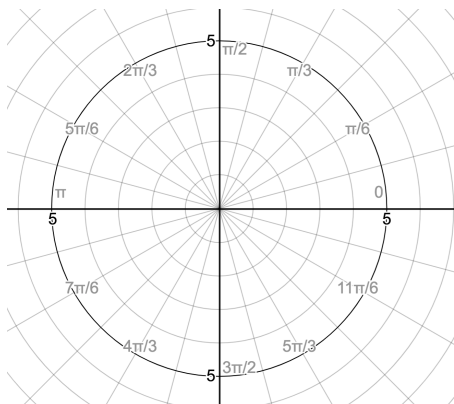
See Polar Cardioid demonstration on Math 8 page and Polar Grapher Geogebra (set view to change)

## CARDIOID

$$r = a + a\cos(\theta) \quad r = a + a\sin(\theta)$$

$$r = a - a\cos(\theta) \quad r = a - a\sin(\theta)$$

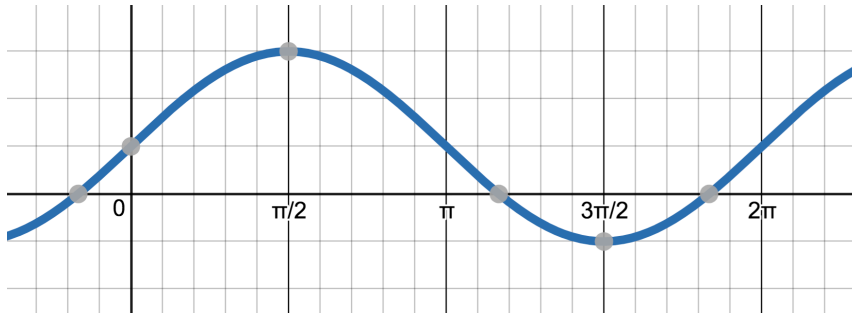
Ex: Graph  $r = 3 + 3\sin(\theta)$  now that we recognize this type of equation.



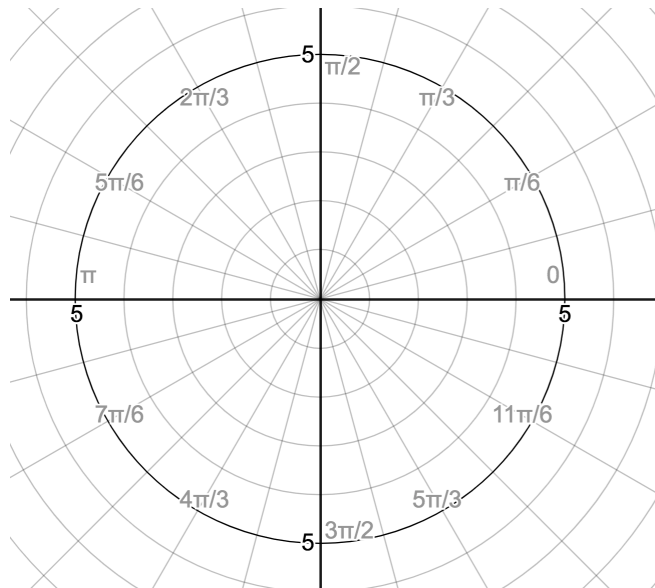
Example:  $r = 1 + 2\sin(\theta)$

Convert: (try it....doesn't help)

Auxiliary Rectangular Graph: (like plotting points but more info and quicker)



Polar Graph:



See Polar Cardioid demonstration on Math 8 page and Polar Grapher Geogebra

LIMACON

$$r = a + b \cos(\theta) \quad r = a + b \sin(\theta)$$

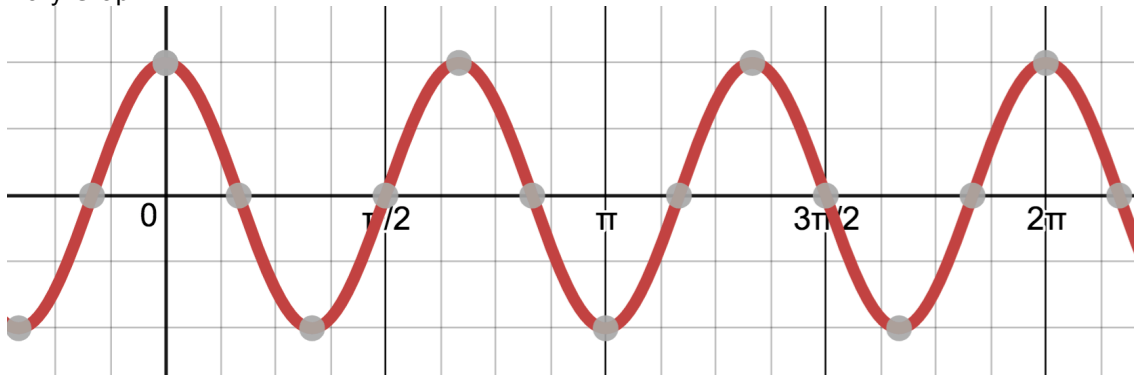
$$r = a - b \cos(\theta) \quad r = a - b \sin(\theta)$$

See Limacon demo, Polar Grapher Geogebra and Desmos Limacon Cardioid on Math 8 page.

Example:  $r = 4 \cos(3\theta)$

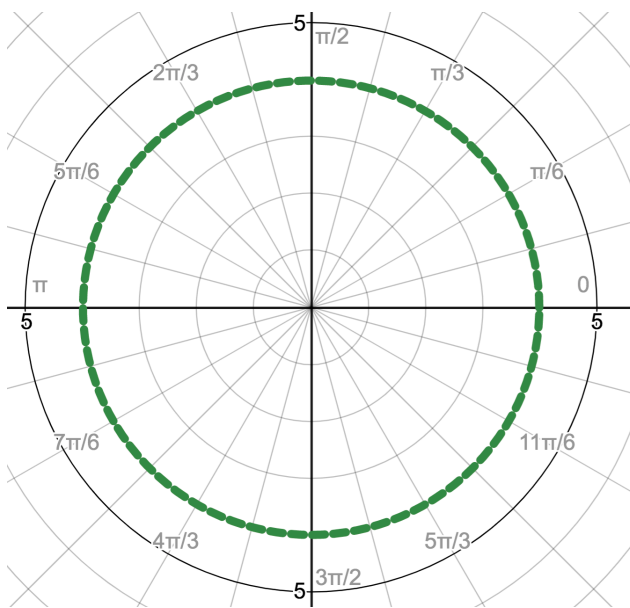
Convert: doesn't help.

Auxilliary Graph:



How would we find the “x intercepts” and highs and lows? What do they correspond to on the polar graph?

Polar Graph



See Polar Grapher geogebra for n even vs n odd.



ROSE

$$r = A \cos(n\theta)$$

$$r = A \sin(n\theta)$$

If we recognize that we have a rose graph, there are shortcuts to finding the graph.

How many petals?  $\left\{ \begin{array}{l} n \text{ odd} \underline{\hspace{2cm}} \\ n \text{ even} \underline{\hspace{2cm}} \end{array} \right.$

How long are the petals?  $\underline{\hspace{2cm}}$

How far apart are the petals spaced?  $\underline{\hspace{2cm}}$

How can we find the location of one petal?  $\underline{\hspace{2cm}}$   
 (Same as finding highs and low of rectangular graph)

Ex: Graph  $r = 5 \sin(2\theta)$  using shortcuts.

