MATH 7B - TEST 1

UNIT 1 – Algebra and Trig. Review plus Applications

This test is in two parts. On part one, you may not use a calculator; on part two, a calculator is necessary. When you complete part one, you turn it in and get part two. Once you have turned in part one, you may not go back to it.

PART ONE - NO CALCULATORS ALLOWED

(1) Find each of the following:

(a) $\cos(30°) =$ $\frac{\sqrt{3}}{2}$

(b) $\csc(3\pi/4) =$ $\sqrt{2}$

(c) $\sec(\pi) =$ -1

(d) $\tan^{-1}(-\sqrt{3}) =$ $-\frac{\pi}{3}$

(e) $\sin^{-1}(-\sqrt{3}/2) =$ $-\frac{\pi}{3}$

(f) $\cos(5\pi/3) =$ $\frac{1}{2}$

(g) $\tan^{-1} 0 =$ 0

(h) $\tan 90° =$ undefined

(i) $\cos^{-1}\left(\frac{-\sqrt{2}}{2}\right) =$ $\frac{3\pi}{4}$

(j) $\sin^{-1}(1) =$ $\frac{\pi}{2}$

(k) $\cos(120°) =$ $-\frac{1}{2}$

(l) $\cos(3\pi) =$ -1

(m) $\sin(9\pi/2) =$ 1

(n) $\cot(5\pi/4) =$ 1

(o) $\tan(-45°) =$ -1

(p) $\sin(315°) =$ $-\frac{\sqrt{2}}{2}$

(q) $\cos\left(\frac{4\pi}{3}\right) =$ $-\frac{1}{2}$

(r) $\cos^{-1}(-1) =$ π

(s) $\sin^{-1}\left(\frac{-\sqrt{2}}{2}\right) =$ $-\frac{\pi}{4}$

(t) $\sin^{-1}(1) =$ $\frac{\pi}{2}$

(2) In what quadrant is each of the following angles?:

(a) $\beta = \cos^{-1}(-1/3)$ II

(b) $\theta = \sin^{-1}(0.2)$ I

(c) $\alpha = \tan^{-1}(-5)$ IV

NAME: ________________

MATH 7B Test 1 - SAMPLE

PART TWO - CALCULATORS ALLOWED (no graphing)

Show your work on this paper. EXACT answers are expected unless otherwise specified.

Fill in the blanks.

In problems 1 - 5 fill in the blank with the most appropriate answer

(1) The range of the function $f(x) = \cos^{-1}x$ is $[0, \pi]$

(2) How many solutions does the equation $\cos x = \frac{1}{4}$ have? infinitely many

(3) How many solutions does the equation $x = \sin^{-1}\left(\frac{1}{5}\right)$ have? Only one

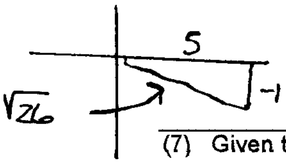
(4) The domain of the function $f(x) = \sin^{-1}x$ is $[-1, 1]$.

(5) $\sin^{-1}(\sin(3\pi/4)) = \pi/4$

(6) Evaluate each of the following exactly:

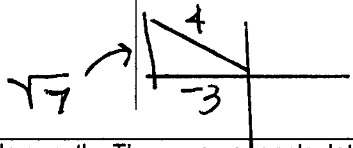
(a) $\cos(\underbrace{\tan^{-1}(-1/5)}_{\theta}) = 5/\sqrt{26}$

$\theta = \tan^{-1}\left(-\frac{1}{5}\right) \Rightarrow \left.\begin{array}{l}\tan\theta = -\frac{1}{5} \\ \text{and} \\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\end{array}\right\} \Rightarrow Q4$

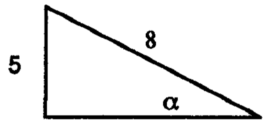


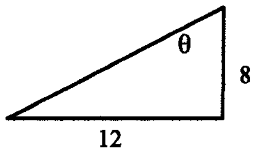
(b) $\tan(\underbrace{\cos^{-1}(-3/4)}_{\theta}) = -\sqrt{7}/3$

$\theta = \cos^{-1}\left(-\frac{3}{4}\right) \Rightarrow \left.\begin{array}{l}\cos\theta = -\frac{3}{4} \\ \text{and} \\ 0 \le \theta \le \pi\end{array}\right\} \rightarrow Q2$



(7) Given the figures below, solve for the variable exactly. Then use your calculator to get an approximation





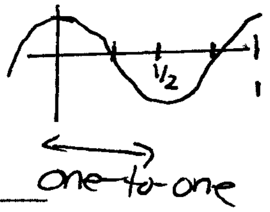
(a) $\alpha = \sin^{-1}\frac{5}{8} \approx 38.7°$

(b) $\theta = \tan^{-1}\frac{12}{8} \approx 56.3°$

(Usually give angle values in degrees, but can answer in radia

(8) How would you restrict the domain of $f(x) = \cos 2\pi x$ in order to make it a one-to-one function? Show how you arrived at that restriction.

$\text{period} = \frac{2\pi}{\text{coeff of } x} = \frac{2\pi}{2\pi} = 1$ so graph would look like



Choose any portion of graph where one to one: $[0, \frac{1}{2}]$

(9) Using your calculator, find approximations for the following, in radians, correct to 3 decimal places.

(a) $\tan^{-1}(5/3) =$ 1.030

(b) $\cos^{-1}(-0.25) =$ 1.823

(d) $\sin^{-1}\left(\frac{\sqrt{2}}{3}\right) =$ 0.491

(10) Solve the following equations exactly. (all solutions)

(a) $\sin\theta = \frac{-\sqrt{2}}{2}$

$\theta = \frac{5\pi}{4} + 2\pi k$

$\frac{7\pi}{4} + 2\pi k$

(b) $\cos x = \frac{\sqrt{3}}{2}$

$x = \begin{cases} \frac{\pi}{6} + 2\pi k \\ \frac{11\pi}{6} + 2\pi k \end{cases}$

(c) $\tan 4\theta = 1$

$4\theta = \frac{\pi}{4} + \pi k$ or $\begin{cases} \frac{\pi}{4} + 2\pi k \\ \frac{5\pi}{4} + 2\pi k \end{cases}$

$\theta = \frac{\pi}{16} + \frac{\pi}{4} k$ $\begin{cases} \frac{\pi}{16} + \frac{\pi}{2} k \\ \frac{5\pi}{16} + \frac{\pi}{2} k \end{cases}$

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(11) Solve the following equations exactly for $0 \le \theta \le 2\pi$. Simplify answers when possible

(a) $\cos\theta = \frac{-1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

(b) $\tan\theta = \frac{\sqrt{3}}{3}$

$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$

(c) $\sin 2\theta = -1$

$2\theta = \frac{3\pi}{2} + 2\pi k$

$\theta = \frac{3\pi}{4} + \pi k$

In $[0, 2\pi)$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

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(12) Solve the following equations exactly for $0 \le \theta \le 2\pi$. Simplify answers when possible

no calculator

(a) $\tan\theta = 8$

$\theta = \tan^{-1} 8$

$\tan^{-1} 8 + \pi$

(b) $\cos\theta = -0.3$

Ref: $= \cos^{-1} 0.3$

$\theta = \begin{cases} \pi - \cos^{-1}(0.3) \\ \pi + \cos^{-1}(0.3) \end{cases}$

(There are other ways to write this)

(c) $\sin\theta = -\frac{1}{6}$

ref $= \sin^{-1}(\frac{1}{6})$

$\theta = \begin{cases} \pi + \sin^{-1}\frac{1}{6} \\ 2\pi - \sin^{-1}\frac{1}{6} \end{cases}$

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(13) Solve the following equations exactly for $0 \le \theta \le 2\pi$. Simplify answers when possible.

(a) $2\sin(\theta) - 1 = 0$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

(b) $2\cos(\theta) - 3 = 6$

$\cos\theta = \frac{9}{2}$.

no solution

(c) $\cot(2\theta) - 1 = 0$

$\cot 2\theta = 1$

$2\theta = \frac{\pi}{4} + \pi k$

$\theta = \frac{\pi}{8} + \frac{\pi}{2} k$

In $[0, 2\pi)$

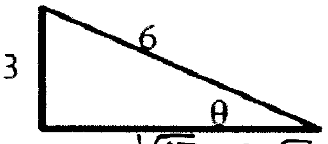
$\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

$x^2+y^2=1$ $\quad x^2=\frac{16}{25}$

$x^2+\frac{9}{25}=1$ $\quad x=\pm\frac{4}{5}$, in Q2

$x=-\frac{4}{5}$

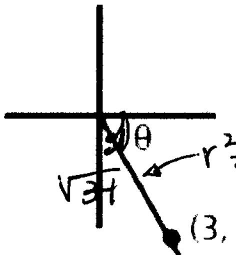
(14) Given the following figures, find:



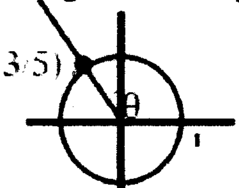
$c^2-a^2=b^2$

$36-9=27$

$\sqrt{27}=3\sqrt{3}$



$\sqrt{34}$ $\quad r^2=x^2+y^2$ $=9+25$



(a) tan θ = $\frac{3}{3\sqrt{3}}=\frac{1}{\sqrt{3}}$ $\quad$ θ ≈ $\frac{\pi}{6}$ or 30°

(b) cos θ = $3/\sqrt{34}$ $\quad$ θ ≈ $-\cos^{-1} 3/\sqrt{34}$

(c) cos θ = $-4/5$ $\quad$ θ ≈ $\cos^{-1}(-4/5)$

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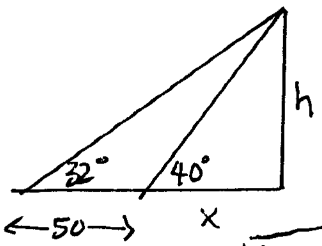
(15) To measure the height of a building, two sightings are taken a distance of 50 feet apart. If the first angle of elevation is 40° and the second is 32°, what is the height of the building (exact and approximate).

Can use law of sines or use two right △s.



small right △ $\quad$ large right △

$\tan 40° = h/x$ $\quad$ $\tan 32° = \frac{h}{x+50}$

$h = x\tan 40°$ $\quad$ $h=(x+50)\tan 32°$

$x\tan 40° = (x+50)\tan 32°$

$\vdots$ $\quad x = \frac{50\tan 32°}{\tan 40° - \tan 32°}$

$h = \frac{50\tan 32°}{\tan 40° - \tan 32°}\tan 40$

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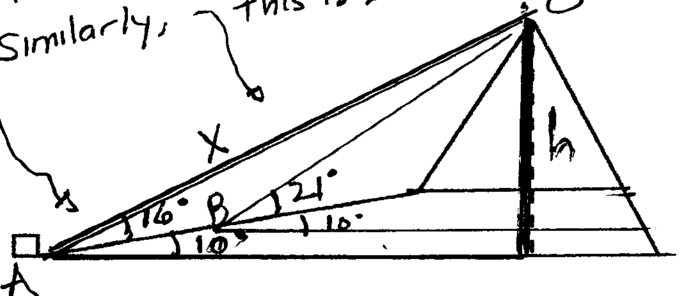
(16) A man observes that the angle of elevation of a mountain peak from his house is 26°. Leaving the house he walks 2000 ft. up a slope of 10° directly towards the mountain and then finds that the angle of elevation of the peak to be 31°. What is the height of the mountain peak (relative to the house). Exact and approximate.

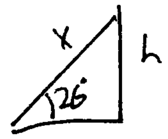
First find angles. Remember angle of elevation measure to horizontal (not to ground)

This angle is 16°, 26° is to horizontal.

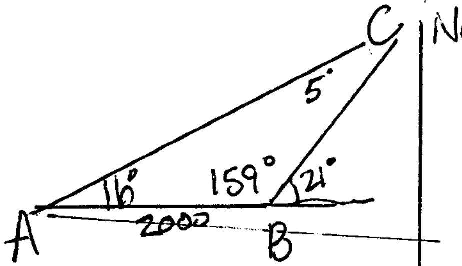
Similarly, this is 21° not 31°



From right △ $\sin 26° = \frac{h}{x}$



so $h = x\sin 26°$



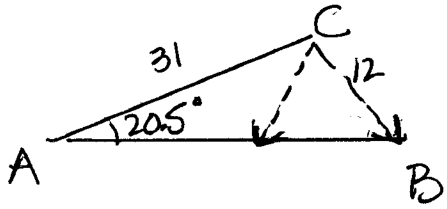
Now using △ABC, find x

$$\frac{x}{\sin 159°} = \frac{2000}{\sin 5°}$$

$$x = \frac{2000\sin 159°}{\sin 5°}$$

so $h = \frac{2000\sin 159°}{\sin 5°}\sin 26° \approx 3605$ ft

(17) Find all remaining parts of the following triangle(s) a=12, b=31, A=20.5° and find the area.



$\frac{\sin B}{31} = \frac{\sin 20.5}{12}$

$\sin B = \frac{31}{12}\sin 20.5^\circ \approx 0.9$

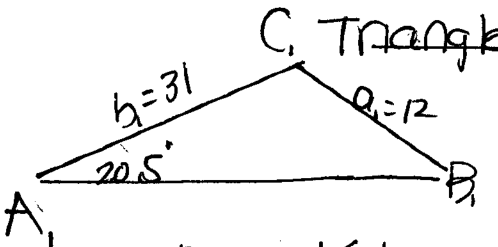
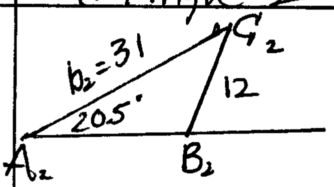
$\frac{c}{\sin C} = \frac{12}{\sin 20.5^\circ}$

$c = \frac{12 \sin C}{\sin 20.5^\circ}$

$\sin B = 0.9$ has two solns in $(0^\circ, 180^\circ)$

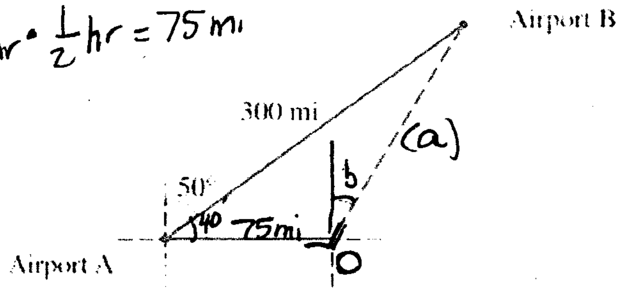
$B_2 \approx 180 - \sin^{-1}(0.9) \approx 115.2^\circ$

$B = \sin^{-1} 0.9 \approx 64.8^\circ$

Triangle 1	Triangle 2
	
$B_1 = \sin^{-1}\left(\frac{31}{12}\sin 20.5^\circ\right) \approx 64.8^\circ$	$B_2 = 180^\circ - B_1 \approx 115.2^\circ$
$C_1 = 180^\circ - A_1 - B_1 \approx 94.7^\circ$	$C_2 = 180^\circ - A - B_2 \approx 44.3^\circ$
$c_1 = 12\sin C_1 / \sin 20.5^\circ \approx 34.15$	$c_2 = 12\sin C_2 / \sin 20.5^\circ \approx 23.9$

(18) Airport B is 300 mi from airport A at a bearing N 50°E (see the figure). A pilot wishing to fly from A to B mistakenly flies due east at 150 mi/h for 30 minutes, when he notices his error.

$150\,mi/hr \cdot \frac{1}{2}hr = 75\,mi$



(a) How far is the pilot from his destination at the time he notices the error? Give your answer correct to the nearest mile.

(b) What bearing should he head his plane in order to arrive at airport B? Give your answer correct to the nearest degree.

a) $a^2 = 300^2 + 75^2 - 2(300)(75)\cos 40^\circ$

$a = \sqrt{95{,}625 - 45{,}000\cos 40^\circ} \approx 247.29$ (store) miles

b) In order to find bearing (angle b), subtract 90° from angle θ = angle AOB.

To find θ: $\frac{\sin\theta}{300} = \frac{\sin 40^\circ}{a}$

$\sin\theta = \frac{300\sin 40^\circ}{a} \Rightarrow \theta = 180 - \sin^{-1}\left(\frac{300\sin 40^\circ}{a}\right) \approx 128.8^\circ \Rightarrow b = \theta - 90^\circ \approx 38.76$

since obtuse.

Ans

N 38.76° E