MATH 7B - TEST 1

UNIT 1 – Algebra and Trig. Review plus Applications

This test is in two parts. On part one, you may not use a calculator; on part two, a calculator is necessary. When you complete part one, you turn it in and get part two. Once you have turned in part one, you may not go back to it.

PART ONE - NO CALCULATORS ALLOWED

(1) Find each of the following:

(a) $\cos(30°) = \frac{\sqrt{3}}{2}$

(b) $\csc(3\pi/4) = \sqrt{2}$

(c) $\sec(\pi) = -1$

(d) $\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$

(e) $\sin^{-1}(-\sqrt{3}/2) = -\frac{\pi}{3}$

(f) $\cos(5\pi/3) = \frac{1}{2}$

(g) $\tan^{-1} 0 = 0$

(h) $\tan 90° =$ undefined

(i) $\cos^{-1}\left(\frac{-\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

(j) $\sin^{-1}(1) = \frac{\pi}{2}$

(k) $\cos(120°) = -\frac{1}{2}$

(l) $\cos(3\pi) = -1$

(m) $\sin(9\pi/2) = 1$

(n) $\cot(5\pi/4) = 1$

(o) $\tan(-45°) = -1$

(p) $\sin(315°) = -\frac{\sqrt{2}}{2}$

(q) $\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$

(r) $\cos^{-1}(-1) = \pi$

(s) $\sin^{-1}\left(\frac{-\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

(t) $\sin^{-1}(1) = \frac{\pi}{2}$

(2) In what quadrant is each of the following angles?:

(a) $\beta = \cos^{-1}(-1/3)$ II

(b) $\theta = \sin^{-1}(0.2)$ I

(c) $\alpha = \tan^{-1}(-5)$ IV

NAME: _____

MATH 7B Test 1 - SAMPLE
PART TWO - CALCULATORS ALLOWED (no graphing)
Show your work on this paper. EXACT answers are expected unless otherwise specified.
Fill in the blanks.

In problems 1 - 5 fill in the blank with the most appropriate answer

(1) The range of the function $f(x) = \cos^{-1}x$ is $[0, \pi]$

(2) The period of $f(x) = \tan 4x$ is $\pi/4$

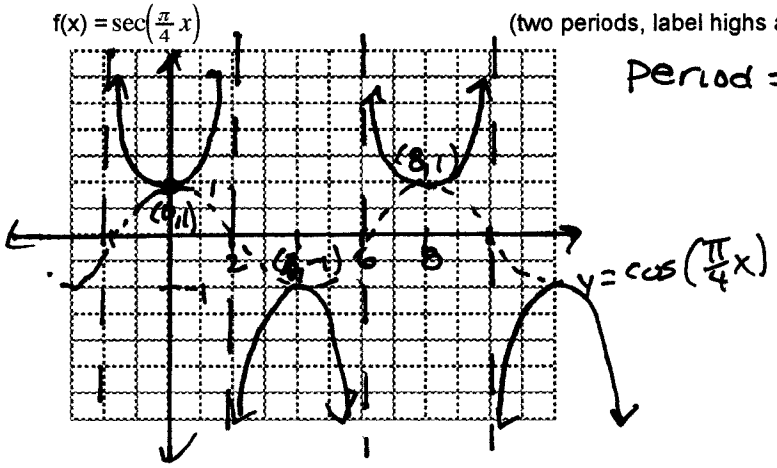
(3) The domain of $f(x) = \cos x$ is $(-\infty, \infty)$

(4) The domain of the function $f(x) = \sin^{-1} x$ is $[-1, 1]$.

(5) $\sin^{-1}(\sin(3\pi/4)) =$ $\pi/4$

(6) Graph the following function. Show work.

$f(x) = \sec\left(\frac{\pi}{4}x\right)$ (two periods, label highs and lows, show scale)



Period $= \frac{2\pi}{\pi/4} = 8$

(7) Solve the following equations exactly. (all solutions)

(a) $\cos\theta = \frac{-\sqrt{3}}{2}$ (b) $\sin\theta = \frac{\sqrt{2}}{2}$ (c) $\tan\theta = 0$

$\theta = \frac{5\pi}{6} + 2\pi k, \frac{7\pi}{6} + 2\pi k$ $\theta = \frac{\pi}{4} + 2\pi k, \frac{3\pi}{4} + 2\pi k$ $\theta = \pi k$

K is an integer

(8) Solve the following equations exactly. $0 \le \theta \le 2\pi$

(a) $\cos\theta = \frac{1}{3}$ (b) $\sin\theta = -1$ (c) $\tan\theta = -5$

$\theta = \cos^{-1}\frac{1}{3}, 2\pi - \cos^{-1}\frac{1}{3}$ $\theta = \frac{3\pi}{2}$ $\theta' = \tan^{-1}5$

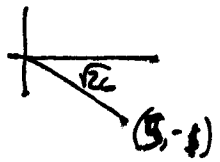
$\pi - \tan^{-1}5 = \theta$

$2\pi - \tan^{-1}5 =$

(9) Evaluate each of the following exactly:

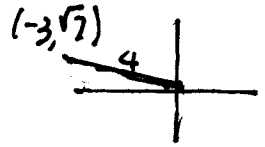
(a) $\cos(\tan^{-1}(-1/5)) =$ $5/\sqrt{26}$

$\theta = \tan^{-1}\left(\frac{-1}{5}\right)$ Q4

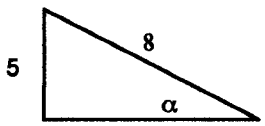


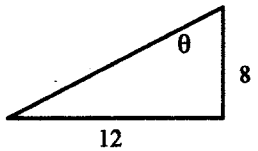
(b) $\tan(\cos^{-1}(-3/4)) =$ $-\sqrt{7}/3$

$\theta = \cos^{-1}\left(-\frac{3}{4}\right)$ Q2



(10) Given the figures below, solve for the variable exactly. Then use your calculator to get an approximation





(a) $\alpha = \sin^{-1}\frac{5}{8} \approx 38.7°$ (b) $\theta = \tan^{-1}\left(\frac{12}{8}\right) \approx 56.3°$

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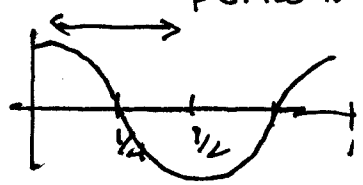
(11) How would you restrict the domain of $f(x) = \cos 2\pi x$ in order to make it a one-to-one function? Show how you arrived at that restriction.



Period is $\frac{2\pi}{2\pi} = 1$

$0 \le x \le 1/2$

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(12) Using your calculator, find approximations for the following ,in radians, correct to 3 decimal places.

(a) $\tan^{-1}(5/3) =$ 1.030 (b) $\cos^{-1}(-0.25) =$ 1.823 (d) $\sin^{-1}\left[\frac{\sqrt{2}}{3}\right] =$ 0.491

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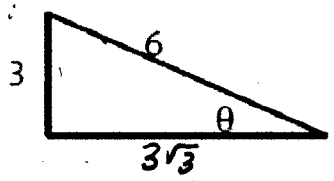
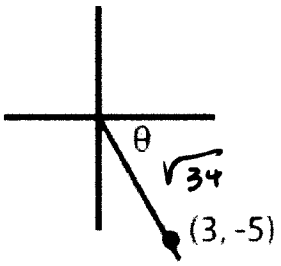
(13) Given the following figures, find:



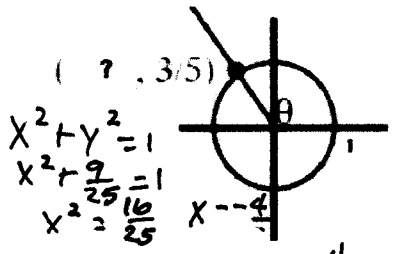
(a) $\tan\theta =$ $1/\sqrt{3}$

$\theta \sim \frac{\pi}{10}$ or 30°



(b) $\cos\theta =$ _____

$\theta \sim -\cos^{-1}\frac{3}{\sqrt{34}}$



(c) $\cos\theta =$ $-\frac{4}{5}$

$\theta \sim \cos^{-1}(-4/5) \approx 143.13°$

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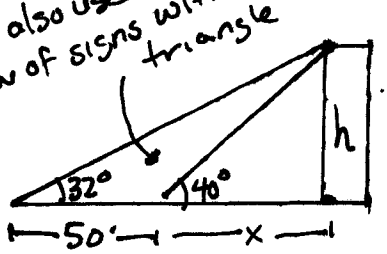
(14) To measure the height of a building, two sightings are taken a distance of 50 feet apart. If the first angle of elevation is 40° and the second is 32°, what is the height of the building (exact and approximate).



can also use Law of Sines with this triangle

small right Δ

$\tan 40° = \frac{h}{x}$

$h = x\tan 40°$

larger right Δ

$\tan 32° = \frac{h}{x+50}$

$(x+50)\tan 32° = h$

$x\tan 40° = (x+50)\tan 32°$

$x\tan 40° = x\tan 32° + 50\tan 32°$

$x\tan 40° - x\tan 32° = 50\tan 32°$

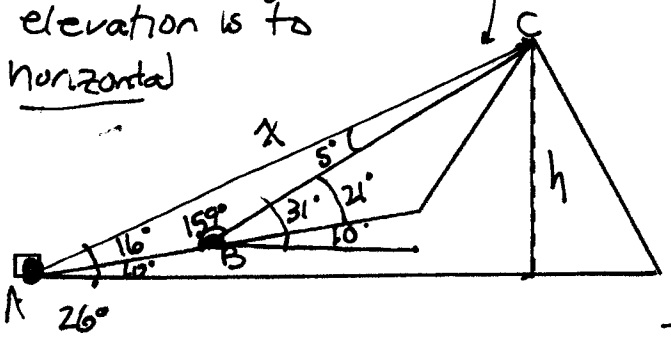
$x = \frac{50\tan 32°}{\tan 40° - \tan 32°}$

$h = \frac{50\tan 32°}{\tan 40° - \tan 32°}\tan 40°$

(15) A man observes that the angle of elevation of a mountain peak from his house is 26°. Leaving the house he walks 2000 ft. up a slope of 10 ° directly towards the mountain and then finds that the angle of elevation of the peak to be 31°. What is the height of the mountain peak (relative to the house). Exact and approximate.

(10 points)

First find angles. Remember angle of elevation is to horizontal



From right triangle $\sin 26° = \frac{h}{x}$

$h = x \sin 26°$

Find X by looking at $\triangle ABC$

$$\frac{X}{\sin 159°} = \frac{2000}{\sin 5°}$$

$$X = \frac{2000}{\sin 5°} \sin 159°$$

Then $h = X \sin 26°$

$$h = \frac{2000}{\sin 5°} \sin 159° \sin 26° \text{ ft}$$

$$\approx 3605 \text{ ft}$$

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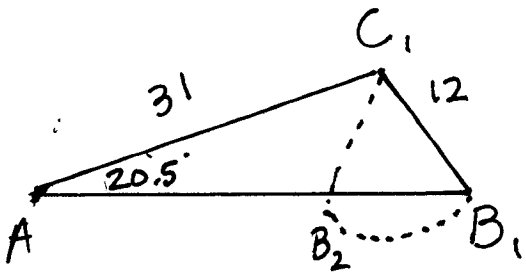
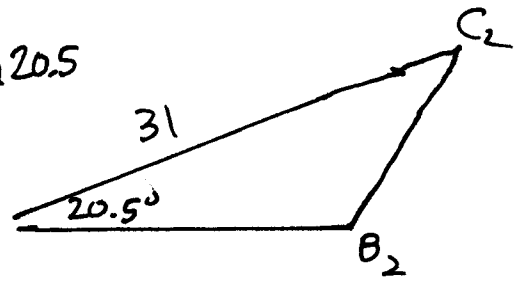
(16) Find all remaining parts of the following triangle(s) a=12, b=31, A=20.5°

$$\frac{\sin B}{31} = \frac{\sin 20.5}{12}$$

$$\sin B = \frac{31}{12} \sin 20.5$$

$\sin B \approx .9$ Two Possible





$B_1 = \sin^{-1}\left(\frac{31}{12} \sin 20.5\right)$

$B_1 \approx 64.8°$

$C_1 = 180° - A - B_1 \approx 94.7°$

$C_1 = \frac{12 \sin C_1}{\sin 20.5} \approx 34.15$

$B_2 = 180° - \sin^{-1}\left(\frac{31}{12} \sin 20.5\right)$

$B_2 \approx 115.2°$

$C_2 = 180° - A - B_2 \approx 44.3°$

$C_2 = \frac{12 \sin C_2}{\sin 20.5°} \approx 23.9$

store and use calculator values

(17) Simplify:

(a) $\dfrac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x}-\frac{1}{y}} \cdot \dfrac{x^2y^2}{x^2y^2}$

$\dfrac{y^2-x^2}{xy^2-yx^2}$

$\dfrac{(y-x)(y+x)}{xy(y-x)} = \dfrac{y+x}{xy}$

(b) $\dfrac{(3+x^2)^{1/2}-x^2(3+x^2)^{-1/2}}{3+x^2}$

$\dfrac{(3+x^2)^{-1/2}(3+x^2-x^2)}{3+x^2}$

$\dfrac{3}{(3+x^2)^{3/2}}$

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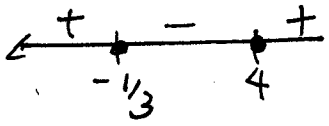
(18) Solve. Express your answer using INTERVAL notation .
Must show appropriate method.

$3x^2-11x-4\le 0$

Quadratic inequality, need sign chart

$(3x+1)(x-4)\le 0$



$[-\frac{1}{3}, 4]$

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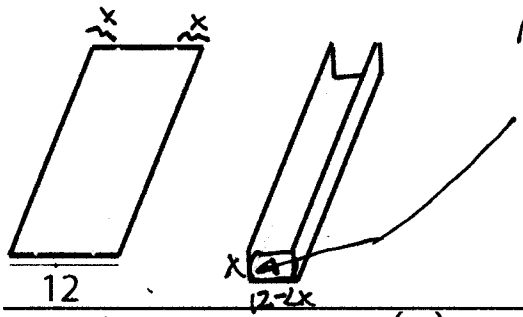
(19) A long, rectangular sheet of metal, 12 inches wide is to be made into a rain gutter by turning up two sides so that they are perpendicular to the sheet. How many inches should be turned up to give the gutter the greatest capacity.



Maximize Capacity by maximizing Area of cross section

$A = x(12-2x)$

$A = 12x-2x^2$

Vertex $x=\frac{-b}{2a}=\frac{-12}{2(-2)}=3$ Turn up 3".

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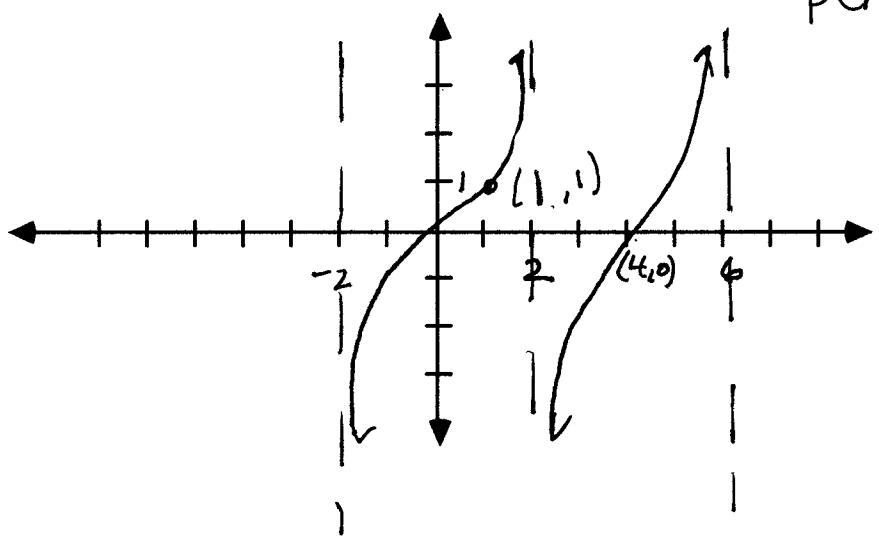
(20) Sketch the graph of f(x) = $\tan\left(\frac{\pi}{4}x\right)$ (TWO periods – show scale clearly)



period = $\pi/\pi_{/4} = 4$

Asymptote when

$\cos\left(\frac{\pi}{4}x\right)=0$

$\frac{\pi}{4}x=\frac{\pi}{2}+\pi k$

$x=\frac{4}{\pi}\left(\frac{\pi}{2}+\pi k\right)$

$x=2+4k$