MATH 7A -TEST : SAMPLE

100 points NAME: Solutions

Show all work on this paper. No credit will be given for solutions if work is not shown (except on the first five problems where it is not necessary to show work). No graphing calculators.

CIRCLE T FOR TRUE, F FOR FALSE. *(2 points each)*

T F (1) The domain of $f(x) = 2^x$ is $(0, \infty)$. $(-\infty, \infty)$

T F (2) $\log_2\left(\sqrt{\frac{y}{4x}}\right)$ can be expanded as $\frac{1}{2}\log_2 y - 1 - \frac{1}{2}\log_2 x$ $\frac{1}{2}\log_2\left(\frac{y}{4x}\right) = \frac{1}{2}(\log_2 y - \log_2 y - \log$

T F (3) $f(x) = \sqrt{x}$ is a one-to-one function.

T F (4) The graph of $f(x) = \log_a(x)$ has an xintercept of 1 for all values of a>0.

T F (5) $f(x) = \frac{1}{x}$ then $f(x) = \frac{1}{x+h}$, not $\frac{1}{x} + h$

Fill in the blanks with the most appropriate answer. *(2 points each)*

(6) $\log_6(36)$ = 2

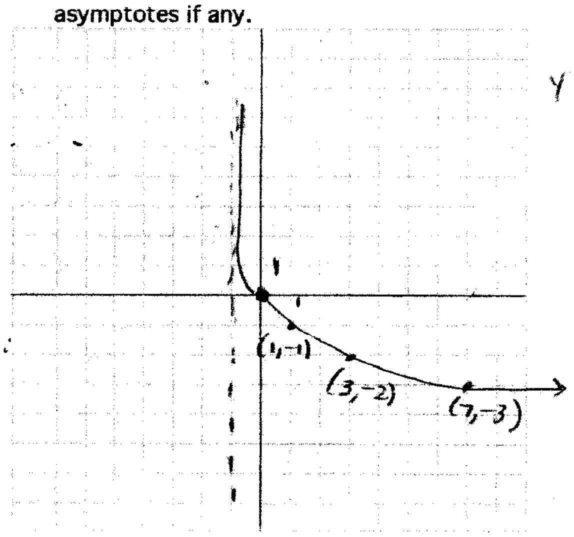
(7) log 0.1= -1

(8) Using your calculator, $\log_2 12$ = $\frac{\ln 12}{\ln 2} \approx 3.585$ (to three decimal places).

(9) $\log_4(-64)$= undefined

(10)) If $f(x) = \sqrt{x-4}$ and $g(x) = \frac{1}{x}$ then $(g \circ f)(x)$ = $\frac{1}{\sqrt{x-4}}$

(11) Sketch the graph of y= $-\log_2(x+1)$ ** LABEL 2 POINTS ON YOUR GRAPH. Show asymptotes if any. *(4 points)*



$y = \log_2 x$ tr left one

a vertical flip

(12) Combine into a single logarithm: $\log_3(a) - \frac{1}{2}\log_3(b) + 7\log_3(c)$ *(4 points)*

$\log_3 a - \log_3 \sqrt{b} + \log_3 c^7$

$\log_3 \frac{ac^7}{\sqrt{b}}$

(13) Solve exactly: $2^{3x-1} = 16$ *(6 points)*

$2^{3x-1} = 2^4$

$3x-1=4$

$3x=5$

$x=\frac{5}{3}$

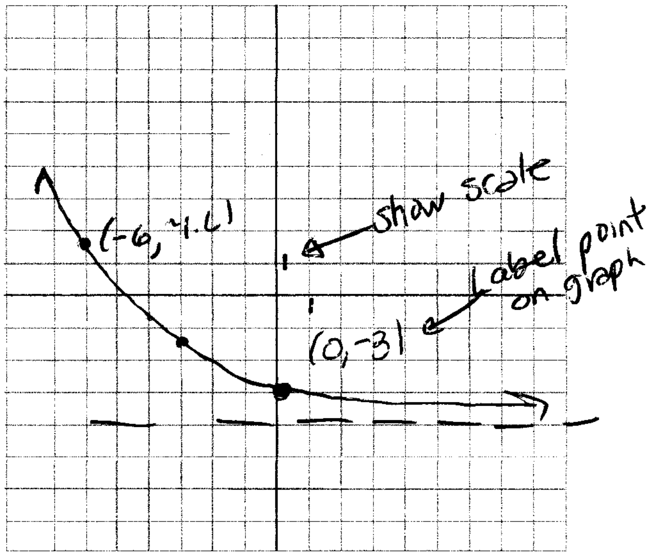
(14) Solve each of the following equations : *(9 points)*

(a) $\log_a 4 = 1/2$ (b) $\log_{27} x = 2/3$ (c) $\log_4(1/16) = A$

$a^{1/2}=4$ $27^{2/3}=x$ $4^A=\frac{1}{16}$

$a=16$ $x=9$ $A=-2$

(15) Sketch the graph of $y=\left(\frac{3}{4}\right)^x - 4$ ** LABEL 2 POINTS ON YOUR GRAPH. Show scale. Show asymptotes if any. *(4 points)*



shift $y=\left(\frac{3}{4}\right)^x$ down 4

Another point :

$x=-3$

$y=\left(\frac{3}{4}\right)^{-3}-4=\left(\frac{4}{3}\right)^3-4$

$=\frac{64}{27}-4=\frac{64-108}{27}$

$=-\frac{44}{27}$

(16) The number of bacteria in a culture is modeled by the function $P(t) = 500\, e^{0.4t}$ where t is measured in hours.

(a) What is the initial number of bacteria?

(b) After how many hours will the number of bacteria reach 5000?

(6 points)

a) $P(0) = 500$ bacteria

b) $5000 = 500\, e^{+0.4t}$

$10 = e^{+0.4t}$

$\ln 10 = +0.4t$

$t = \dfrac{\ln 10}{0.4} \approx 5.756$ hrs

---

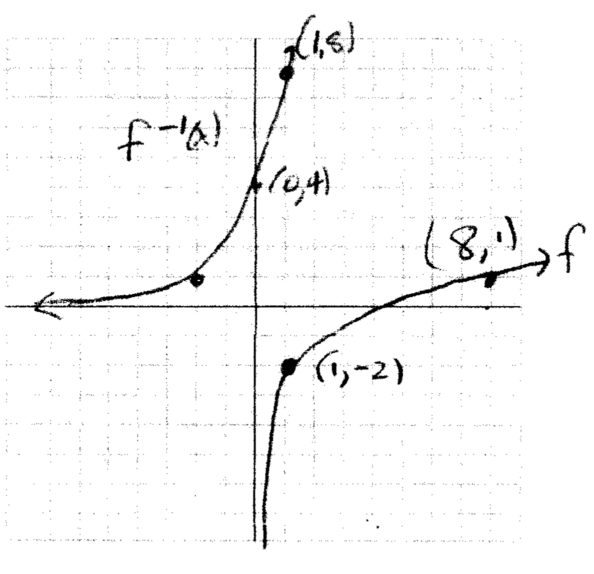
(17) Given $f(x) = \log_2(x) - 2$

(a) find $f^{-1}(x)$.

(b) Graph $f(x)$ and $f^{-1}(x)$. Label each graph and label one point on each graph.

(c) Find the domain and range for $f(x)$ and for $f^{-1}(x)$.

(15 points)



$y = \log_2(x) - 2$

switch

$x = \log_2(y) - 2$

$x + 2 = \log_2(y)$

$2^{x+2} = y$

$f^{-1}(x) = 2^{x+2}$

	f	f^{-1}
domain	$(0,\infty)$	$(-\infty,\infty)$
range	$(-\infty,\infty)$	$(0,\infty)$

(18) Solve the following logarithmic equations. *(6 points each)*

(a) $\log x + \log(x+3) = \log 18$

$\log(x(x+3)) = \log 18$

$x^2 + 3x = 18$

$x^2 + 3x - 18 = 0$

$(x+6)(x-3) = 0$

$x = \cancel{-6}, 3$

Must check

$x = 3$

(b) $\log_3(x-1) - \log_3(x+2) = 2$

$\log_3 \frac{x-1}{x+2} = 2$

$\frac{x-1}{x+2} = 9$

$x - 1 = 9x + 18$

$-19 = 8x$

$x = -19/8$

does not check - no solution

---

(19) Solve the following exponential equations. (Find the exact solutions, then get an approximation using your calculator) *(6 points each)*

(a) $2^{3-2x} = 7$

$\ln(2^{3-2x}) = \ln 7$

$(3-2x)\ln 2 = \ln 7$

$3 - 2x = \frac{\ln 7}{\ln 2}$

$-2x = \frac{\ln 7}{\ln 2} - 3$

$x = -\frac{1}{2}\frac{\ln 7}{\ln 2} + \frac{3}{2} \approx .096$

(b) $5^{x+4} = 3^x$

$\ln 5^{x+4} = \ln 3^x$

$(x+4)\ln 5 = x \ln 3$

$x\ln 5 + 4\ln 5 = x\ln 3$

$4\ln 5 = x\ln 3 - x\ln 5$

$4\ln 5 = x(\ln 3 - \ln 5)$

$x = \frac{4\ln 5}{\ln 3 - \ln 5} \approx -12.603$

---

(20) Given $f(x) = -\sqrt{x-2}$ find the following. Pay close attention to details, show all steps precisely.

(a) $f^{-1}(x)$ (5 points)

(d) Verify $(f \circ f^{-1})(x) = x$. (3 points)

$y = -\sqrt{x-2}$

switch

$x = -\sqrt{y-2}$

$x^2 = y - 2$

caution... introduced extra solns

$y = x^2 + 2$

$f^{-1}(x) = x^2 + 2;\ x \leq 0$

$(f \circ f^{-1})(x)$

$= f(x^2+2)$

$x \leq 0$

$= -\sqrt{x^2+2-2}$

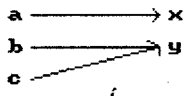
$= -\sqrt{x^2} = -|x|$

But $|x| = -x$ since $x \leq 0$

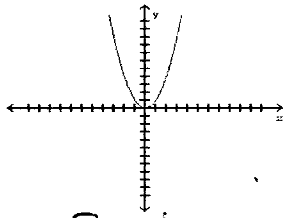
$= -(-x) = x$

(21) Determine whether each of the following are best described as relations, functions, or one to one functions.

(a) $\{(4,0), (4,3), (-3,7)\}$ relation (b) $y^2 = x$ relation (c) $f(x) = 3x-1$ one-to-one function

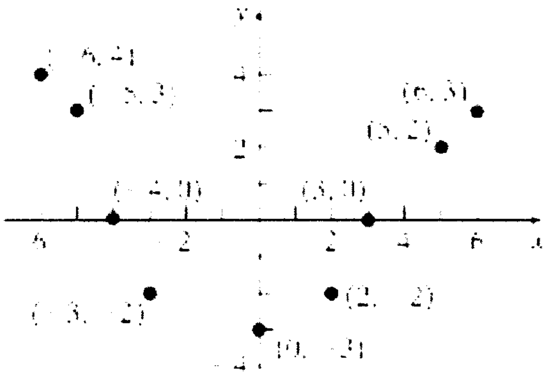


(d) function



(e) function

(22) Using the graph of f(x) below, find



(a) f(-3) -2

(b) f(0) -3

(c) For what values of x is f(x)<0 (-4, 3)

(d) What are the zeros of f? -4, 3

(e) For what number(s) x does f(x)=3? -5, 6

(e) What is the y intercept of f? -3

(f) Domain of f: [-6, 6]

(g) Range of f: [-3, 4]

(h) How many times does the graph of y=1 intersect this graph? 2

(i) Explain why a function can have at most one y intercept? if it had two, say (0,1) and (0,2) then x=0 would have 2 different y values - not a function

(23) Given the graph of y = f(x) as shown on both graphs below, Find:

Use the graph of f(x) to graph each of he following. Label two points on your graph.

(a) y = 2f(x) vertical stretch (b) y = f(2x). horizontal shrink