**Math Path 5C test 2 – Chapter 13**
**Sample – In class portion**

50 points Name: Solns

(1) Find the length of the curve $\vec{r}(t) = \langle 2t^{3/2}, \cos 2t, \sin 2t \rangle$ for $0 \le t \le 4$.

$\vec{r}\,' = \langle 3t^{1/2}, -2\sin 2t, 2\cos 2t \rangle \quad \|\vec{r}\,'\| = \sqrt{9t+4}$

$L = \int_0^4 \sqrt{9t+4}\,dt = \frac{1}{9}\frac{2}{3}(9t+4)^{3/2}\Big]_0^4 = \underline{\frac{2}{27}\left(40^{3/2} - 8\right)}$

(2) Find the point of intersection, if any, of the helix $\mathbf{r}_1(t) = \langle \cos t, \sin t, t \rangle$ and the curve $\mathbf{r}_2(t) = \langle 1+t, t^2, t^3 \rangle$. Find the equations of the tangent lines to each of the curves at this point.

$$\begin{array}{l} x \\ y \\ z \end{array} \left\{ \begin{array}{l} \cos t = 1+t \\ \sin t = t^2 \\ t = t^3 \Rightarrow t = 0, 1, -1 \end{array} \right\} \Rightarrow t = 0$$

intersection point $(1,0,0)$

For $\vec{r}_1$: $\vec{r}_1\,'(t) = \langle -\sin t, \cos t, 1 \rangle$

$\vec{v}_1 = \vec{r}_1\,'(0) = \langle 0, 1, 1 \rangle$

tangent line $\left\{ \begin{array}{l} x = 1 \\ y = t \\ z = t \end{array} \right.$

For $\vec{r}_2$: $\vec{r}_2\,'(t) = \langle 1, 2t, 3t^2 \rangle \quad \vec{v}_2 = \vec{r}_2\,'(0) = \langle 1, 0, 0 \rangle$

tangent line $\left\{ \begin{array}{l} x = 1+t \\ y = 0 \\ z = 0 \end{array} \right.$

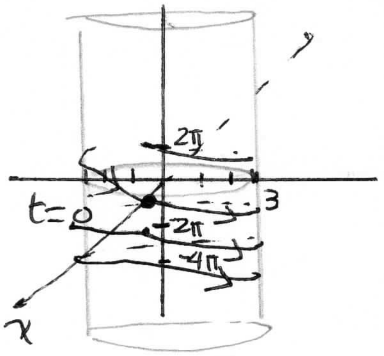
(3) Sketch the graph of $\vec{r}(t) = \langle \cos t, 3\sin t, -t \rangle$, and show direction of increasing t. Give the equation of a surface on which this curve lies and show this surface on your sketch. *(10 points)*

$\left. \begin{array}{l} x = \cos t \\ y = 3\sin t \end{array} \right\}$

$z = -t$

$\left. \begin{array}{l} x = \cos t \\ \frac{y}{3} = \sin t \end{array} \right\}$ square and add.

$x^2 + \frac{y^2}{9} = 1$ elliptical cylinder




(4) Let C be the curve with equations $x = t^2 - 2$, $y = -\ln t$, $z = \cos(\pi t/2)$. Find the point where C intersects the xz plane and find the equation of the normal plane to C at that point. *(10 points)*

C intersects xz plane when $y = 0 \Rightarrow -\ln t = 0 \Rightarrow t = 1$ $\quad$ point $(-1, 0, 0)$

$\vec{r}\,' = \langle 2t, -\frac{1}{t}, -\frac{\pi}{2}\sin\frac{\pi t}{2} \rangle$

$\vec{r}\,'(1)$ is the normal vector $\vec{n}$ for the normal plane $\vec{n} = \langle 2, -1, -\frac{\pi}{2} \rangle$

$$\underline{2(x+1) - y - \frac{\pi}{2}z = 0}$$

From 14.1 and 14.2

(1) The following limit exists. Find the value:

$\lim_{(x,y)\to(1,4)} \frac{4x-y}{16x^2-y^2}$ ____$\frac{1}{8}$____  $\frac{4x-y}{(4x-y)(4x+y)} = \frac{1}{4x+y}$

(2) For the function $f(x,y) = \frac{x^3y}{3x^6+y^2}$,

(a) Find $\lim_{(x,y)\to(0,0)} f(x,y)$ approaching along any straight line y = mx.( $m \neq 0$) ____0____

$\lim_{x\to 0} \frac{x^3\, mx}{3x^6+m^2x^2} = \lim_{x\to 0} \frac{mx^4}{3x^6+m^2x^2} = \lim_{x\to 0} \frac{mx^2}{3x^4+m^2} = \frac{0}{m^2} = 0$

(*) → make sure you are doing correct work to get zero.

(b) Find $\lim_{(x,y)\to(0,0)} f(x,y)$ along the curve $y=x^2$.____0____(similarly)

(c) Find $\lim_{(x,y)\to(0,0)} f(x,y)$ along the curve $y = x^3$.____$\frac{1}{4}$____

→ $\lim_{x\to 0} \frac{x^3x^3}{3x^6+x^6} = \lim_{x\to 0} \frac{x^6}{4x^6} = \lim_{x\to 0} \frac{1}{4} = \frac{1}{4}$

(d) What can be said about $\lim_{(x,y)\to(0,0)} f(x,y)$? DNE

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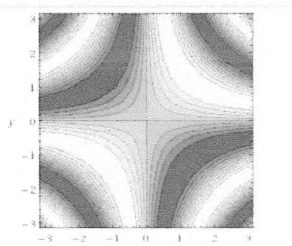
(3) Match the following functions with their level curves (a-d) and their graphs (1-4):

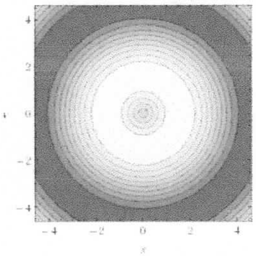
$f(x,y) = \sin(\sqrt{x^2+y^2})$ ____4____ ____b____  $f(x,y) = \sin(xy)$ ____3____ ____a____

$f(x,y) = |x+y|$ ____2____ ____c____  $f(x,y) = |xy|$ ____1____ ____d____

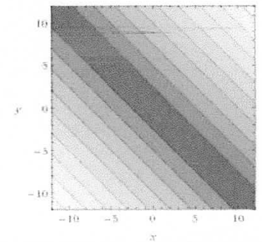
a)



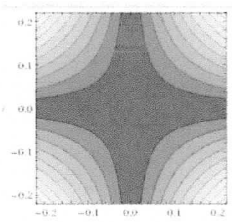
b)



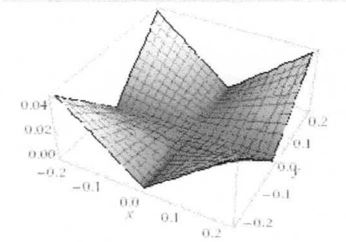
c)



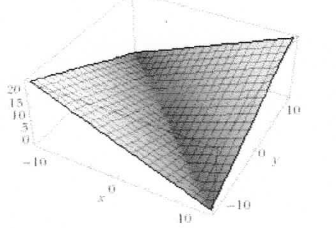
d)



(1)

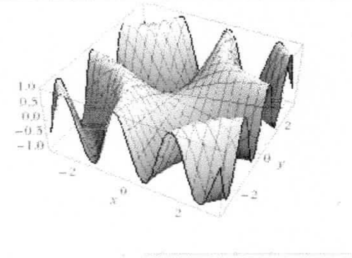


(2)



(3)

3D plot



(4)