

100 points

NAME: _____

Be sure to show work neatly and follow instructions carefully. Name any theorems used.

- (1) Given the vector field $\vec{F}(x,y) = (4x + y, x + 2y)$ and the curve C: $y = 1 - x^2$ from (0,1) to (1,0),
- Compute the line integral of \vec{F} along C directly, using the definition.
 - Is \vec{F} a gradient field? If so find $f(x,y)$ such that $\vec{F} = \nabla f(x,y)$ and use it to verify your answer in part (a).
 - Does Green's theorem apply to this line integral? If so, evaluate the line integral using Green's theorem, if not, why not?
 - For \vec{F} as above but C is now the square with vertices (0,0), (1,0), (1,1), (0,1) oriented counter-clockwise, compute $\int_C \vec{F} \cdot d\vec{r}$
- (2) Given C is the piecewise smooth curve from (4,0) to (4,4) to (0,0) and back to (4,0) find $\int_C y^2 dx + x^2 dy$ using two different methods.
- (3) Evaluate $\iint_S (x^2 + y^2 + z^2) dS$ where S is the portion of the cylinder $x^2 + y^2 = 9$ in the first octant that lies below the plane $z=y$.
- (4) Given $\vec{F}(x,y,z) = xz \mathbf{i} + 2xy \mathbf{j} + 3xy \mathbf{k}$, and C is the boundary of the part of the plane $3x + y + z = 3$ in the first octant, oriented clockwise as viewed from above, evaluate $\int_C \vec{F} \cdot d\vec{r}$ TWO ways, (a) directly and (b) using another method. (DETAILS)
- (5) Given: $\vec{F}(x,y,z) = \langle y, x, z \rangle$ and S is the closed surface formed by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$ and is oriented by outward unit normals, evaluate $\iint_S \vec{F} \cdot d\vec{S}$ TWO ways: (a) directly, and (b) using another method (DETAILS).