

100 points

NAME: _____

Be sure to show work neatly and follow instructions carefully.

Longer than actual exam

(1) Evaluate $\iint_R \frac{xy}{\sqrt{x^2 + y^2 + 1}} dA$ $R: \{(x,y): 0 \leq x \leq 1, 0 \leq y \leq 1\}$.

Ans: $\frac{1}{3} - \frac{4\sqrt{2}}{3} + \sqrt{3}$

(2) SET UP BUT DO NOT EVALUATE: Use a double integral in polar coordinates to find the volume of the solid bounded above by the surface $z = 1 - x^2 - y^2$, below by the xy plane, and laterally by the cylinder $x^2 + y^2 - x = 0$.

Ans: $\int_{-\pi/2}^{\pi/2} \int_0^{\cos\theta} (1-r^2)r dr d\theta$

(3) Evaluate $\int_0^2 \int_{y/2}^1 \cos(x^2) dx dy$

Ans: $\sin 1$

(4) (a) Convert $(4, \pi/2, 3)$ from cylindrical coordinates to

rectangular coordinates _____ Ans: $(0, 4, 3)$

spherical coordinates _____ Ans: $(5, \pi/2, \cos^{-1}(3/5))$

(b) Convert $(2, 2, \sqrt{2})$ from rectangular coordinates to

cylindrical coordinates _____ Ans: $(2\sqrt{2}, \pi/4, \sqrt{2})$

spherical coordinates _____ Ans: $(\sqrt{10}, \pi/4, \cos^{-1}(1/\sqrt{5}))$

(5) SET UP BUT DO NOT EVALUATE: $\iiint_E f(x,y,z) dV$ where E is the solid bounded by the

paraboloid $z = 4x^2 + y^2$ and the parabolic cylinder $z = 4 - 3y^2$.

Ans: $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{4x^2+y^2}^{4-3y^2} f(x,y,z) dz dy dx$ (many others possible)

- (6) SET UP BUT DO NOT EVALUATE: integrals as specified to find the volume enclosed by the cone $z^2 = x^2 + y^2$ and the plane $z = 4$.

a) Triple integral - cylindrical coordinates.

$$\text{Ans: } \int_0^{2\pi} \int_0^4 \int_r^4 r \, dz \, dr \, d\theta$$

b) Triple integral - spherical coordinates.

$$\text{Ans: } \int_0^{2\pi} \int_0^{\pi/4} \int_0^{4 \sec \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

c) Double integral - rectangular coordinates ; order dy dx .

$$\text{Ans: } 4 \int_0^4 \int_0^{\sqrt{16-x^2}} (4 - \sqrt{x^2 + y^2}) \, dy \, dx$$

d) Triple integral- rectangular coordinates; order dx dz dy

$$\text{Ans: } 4 \int_0^4 \int_y^4 \int_0^{\sqrt{z^2 - y^2}} \, dx \, dz \, dy$$

(7) Given $\int_0^{2\pi} \int_0^1 \int_0^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta$

a) The above integral can be used to compute the volume of a solid. Sketch the solid.

Ans: It is a cylinder of radius one topped by a portion of a sphere of radius 2.

b) Convert the triple integral to Rectangular Coordinates: DO NOT EVALUATE.

$$\text{Ans. } 4 \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{4-x^2-y^2}} \, dz \, dy \, dx$$

c) Convert the triple integral to Spherical Coordinates: DO NOT EVALUATE.

Ans: This one is a little difficult, must be split into two solids

$$\int_0^{2\pi} \int_0^{\pi/6} \int_0^2 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta + \int_0^{2\pi} \int_{\pi/6}^{\pi/2} \int_0^{\csc \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

(8) Evaluate $\iint_S z \, dS$ where S is the portion of the paraboloid $z=x^2 + y^2$ that lies under the plane

$$z=4.$$

$$\text{Ans: } \frac{\pi}{60} (391\sqrt{17} + 1)$$

(9) A lamina the shape of a quarter circle of radius 4 has density proportional to the distance from the origin. Find the center of mass. Ans: $\left(\frac{6}{\pi}, \frac{6}{\pi}\right)$

(10) $\int_C yz \cos x \, ds$ where C is given by $x=t, y=2\cos t, z=2\sin t, 0 \leq t \leq \pi$. Ans: $\frac{8\sqrt{5}}{3}$