MATH 5C – SAMPLE FINAL EXAM

(1) The position vector of a particle is $\mathbf{r}(t) = e^t\,\mathbf{i} + t\,\mathbf{j}$, $t \geq 0$.

(a) Find T .

(b) Sketch the graph of $\mathbf{r}(t)$ showing direction of increasing t.

(c) Find the curvature.

(2) Evaluate the integral expression:

$$\int_0^3 \int_{y^2}^9 y\cos\left(x^2\right)dx\,dy$$

(Hint: You *may* want to try reversing the order of integration, although it is not necessary)

(3) Find parametric equations of the line that passes through the point of intersection of L1 and L2 and is orthogonal to both L1 and L2 where L1 and L2 are the lines given by:

$$L_1 \begin{cases} x = 2t-1 \\ y = 1-t \\ z = 3t \end{cases} \qquad L_2 \begin{cases} x = 1+s \\ y = 2s \\ z = 3-2s \end{cases}$$

(4) Find the point of intersection (if any) of the tangent lines to the curve $\mathbf{r}(t) = \langle \cos t, \cos t, \sqrt{2}\sin t \rangle$ at the points where t = 0 and t= π/4

(5) SET UP BUT DO NOT EVALUATE: integrals as specified to find the volume of the wedge in the first octant cut from the cylinder $x^2+y^2=4$, the xy plane, and the plane z=y.

a) Show sketch.

b) Triple integral - dx first.

c) Triple integral - dy first.

d) Double integral.

(6) Given $f(x,y) = x^2 e^y$,

a) Find $D_u f(-2,0)$ in the direction of $\vec{a} = \langle 4, 3 \rangle$.

b) Find the maximum value of the directional derivative at (-2,0) and the direction in which it occurs.

(7) SET UP BUT DO NOT EVALUATE: integrals as specified to find the volume enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 8$.

a) Sketch the solid

b) Triple integral - cylindrical coordinates.

c) Triple integral - spherical coordinates.

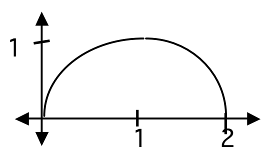
d) Triple integral - rectangular coordinates .

e) Now actually find the volume.

(8) Find the dimensions of a rectangular box, open at the top, having a volume of 108 ft^3 and requiring the least amount of material for its construction.

(9) Given the vector field $\vec{F}(x,y) = \langle y^3+1, 3xy^2+1 \rangle$ and the semicircular path C given by $\vec{r}(t) = (1 - \cos t)\,\vec{i} + \sin t\,\vec{j}$, $0 \le t \le \pi$ as shown,

a) Show that $\vec{F}$ is a conservative vector field.



b) Find the potential function f(x,y) such that $\vec{\nabla} f = \vec{F}$.

c) Find $\int_C \vec{F} \bullet d\vec{r}$ using f.

d) Find $\int_C \vec{F} \bullet d\vec{r}$ using a different method.

(10) Given $\vec{F}(x,y,z) = (x - y)\,\vec{i} + (y - z)\,\vec{j} + (z - x)\,\vec{k}$ and C is the intersection of the paraboloid $z=x^2+y^2$ and the sphere $x^2+y^2+z^2=2$, counter clockwise when viewed from above, find $\int_C \vec{F} \bullet d\vec{r}$ two ways:

(a) directly, and
(b) using an appropriate theorem.

ANSWERS:

(1) (a) $\left\langle \frac{e^t}{\sqrt{e^{2t}+1}}, \frac{1}{\sqrt{e^{2t}+1}} \right\rangle$ (b) graph of y=ln(x) from point (1,0) to right (c) $\kappa = \frac{e^t}{\left(e^{2t}+1\right)^{3/2}}$

(2) ¼ sin 81 (3) x=1-4t, y=7t, z=3+5t (4) (1, 1, 2 - $\sqrt{2}$)

(5) b) $\int_0^2 \int_0^y \int_0^{\sqrt{4-y^2}} dx\, dz\, dy$

c) $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_z^{\sqrt{4-x^2}} dy\, dz\, dx$

d) $\int_0^2 \int_0^{\sqrt{4-x^2}} y\, dy dx$

(6) (a) -4/5 (b) In direction of gradient <-4,4>, value 4 $\sqrt{2}$).

(7) b) $\int_0^{2\pi} \int_0^2 \int_r^{\sqrt{8-r^2}} r\ dz dr d\theta$

c) $\int_0^{2\pi} \int_0^{\pi/4} \int_0^{\sqrt{8}} \rho^2 \sin\phi\ d\rho d\phi d\theta$

d) $4\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} dz\, dy\, dx$

e) $\frac{32\pi}{3}\left(\sqrt{2}-1\right)$

(8) Minimize 2yz+2xz+xy subject to xyz=108. Ans: 6ft by 6 ft by 3 ft high.

(9) (b) f(x,y) $xy^3+x+y+C$ (c) 2 (d) 2

(10) π