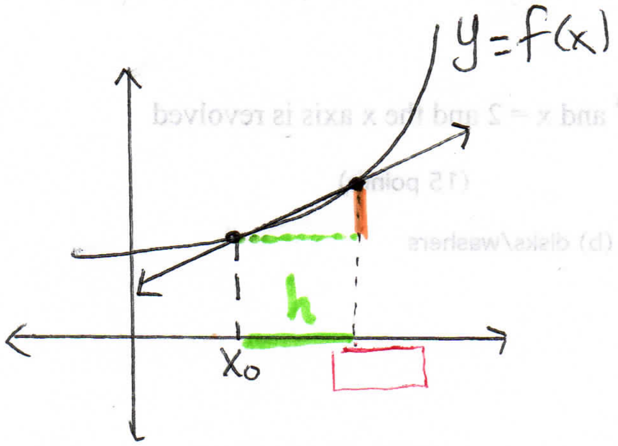
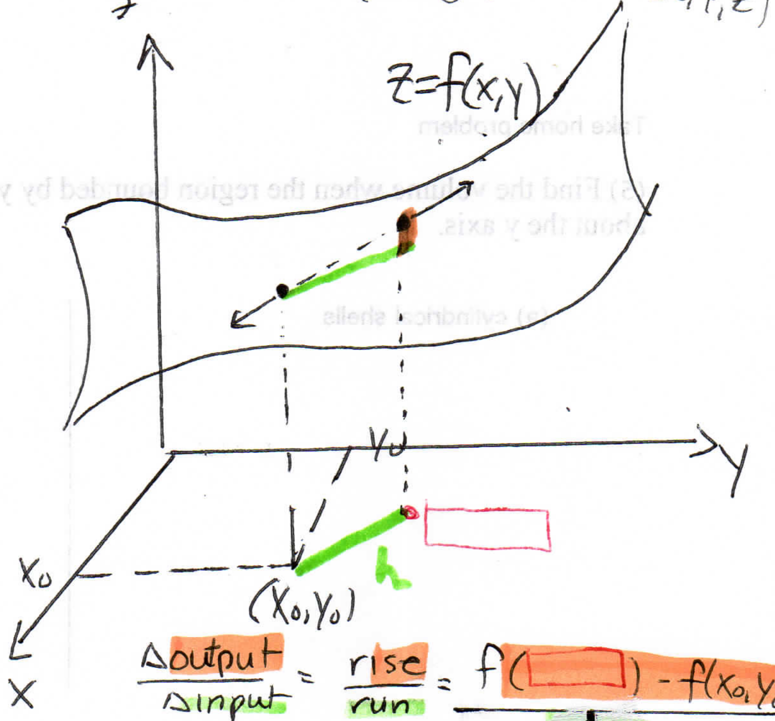
Derivative

(extends to $f(x,y,z)$)



Average rate of change "$\dfrac{\Delta\text{output}}{\Delta\text{input}} = \dfrac{\text{rise}}{\text{run}}$"

$$f'(x_0)=\lim_{h\to 0}\frac{f(\boxed{})-f(x_0)}{h}$$

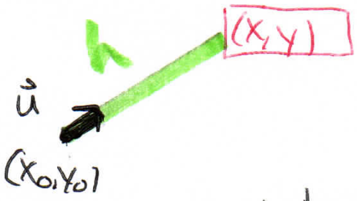


$$\frac{\Delta\text{output}}{\Delta\text{input}}=\frac{\text{rise}}{\text{run}}=\frac{f(\boxed{})-f(x_0,y_0)}{h}$$

and "derivative" would be lim as $h\to 0$

What is the new input? $\boxed{}$

Let the direction we move from (x_0,y_0) be specified by a <u>unit vector</u> $\langle a,b\rangle$. We want to find the new input, call it (x,y).



Form vector from (x_0,y_0) to (x,y)

$\langle x-x_0, y-y_0\rangle$

and note $\langle x-x_0, y-y_0\rangle = h\vec{u}$

$\langle x-x_0, y-y_0\rangle = h\langle a,b\rangle$

$\Rightarrow x-x_0=ah,\ y-y_0=bh$

$\Rightarrow x=x_0+ah,\ y=y_0+bh$

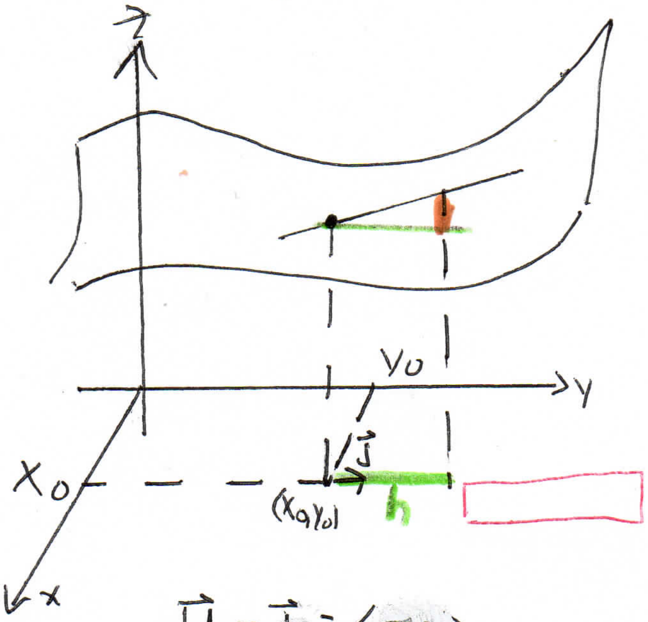
$\Rightarrow$ new input $\boxed{(x_0+ah, y_0+bh)}$

Then the derivative of $f(x,y)$ in the direction of $\vec{u}=\langle a,b\rangle$ at the point (x_0,y_0) is

$$\lim_{h\to 0}\frac{f(x_0+ah,y_0+bh)-f(x_0,y_0)}{h} = \boxed{}$$

Special cases of $D_{\vec{u}}f(x_0, y_0)$

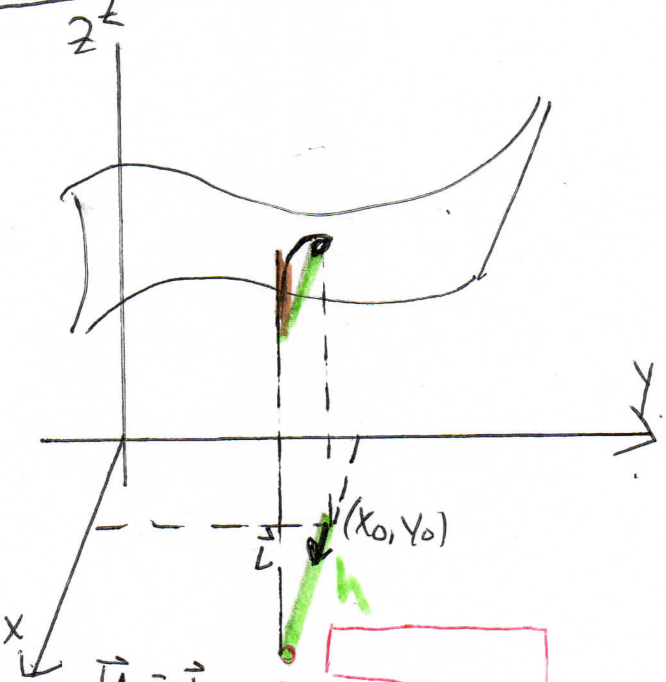
(note: book uses (a,b) instead of (x_0, y_0) but we use $\langle a,b\rangle$ for $\vec{u}$):



$\vec{u} = \vec{j} = \langle 0,1\rangle$

move parallel to y-axis in positive y direction

$$D_{\vec{u}}f(x_0,y_0) = \lim_{h\to 0} \frac{f(\boxed{}) - f(x_0,y_0)}{h}$$



$\vec{u} = \vec{i} = \langle 1,0\rangle$

move parallel to x-axis in positive x direction

$$D_{\vec{u}}f(x_0,y_0) = \lim_{h\to 0} \frac{f(\boxed{}) - f(x_0,y_0)}{h}$$

Goal is still to let $\vec{u}$ be in <u>any</u> direction