(1) F could be discontinuity or Vertical tangent

(2) T (3) F (4) F $dy = 5\cos 5x\,dx$ (5) F Could be at an endpoint

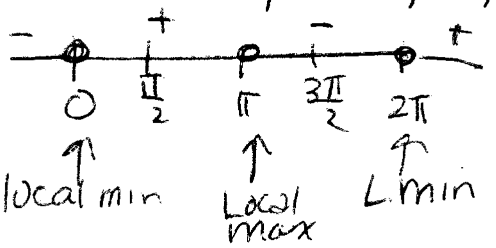
(6) pg 205 (8th edition

(7) $f(x) = \sin^2 x - 2\cos x$

a) $f'(x) = 2\sin x\cos x + 2\sin x = 2\sin x(\cos x + 1) \overset{set}{=} 0$

$x = \pi k$, k integer

b) Consider one period, $[0, 2\pi)$



local min at $x = 2\pi k$ value -2

local max at $x = \pi + 2\pi k$ value 2

crit #s / end pts

x	π	2π	0	3π
$f(x)$	2	−2	−2	2

c) Closed interval method

Abs min is -2

Abs max is 2

(8) a) $f'(x) = \frac{2}{3x^{1/3}}$ which is not diffable at $x=0$ so Rolles does not apply to f on $[-1,1]$

b) $f(x)$ is conts & diffable on $(-\infty, \infty)$ so MVT applies on $[0,2]$. ∴ There must exist $c \in (0,2)$ s.t.

$f'(c) = \frac{f(2) - f(0)}{2-0} \Rightarrow 3c^2 - 1 = \frac{6-0}{2-0} \Rightarrow 3c^2 - 1 = 3$

$3c^2 = 4$

$c = \pm\sqrt{\frac{4}{3}}$ In $[0,2]$ $\boxed{c = \sqrt{\frac{4}{3}}}$

(9) maximize AND minimize distance from (x,y) on $y = \sqrt{x}$, $0 \le x \le 9$ to $(2,0)$

dist $= \sqrt{(x-2)^2 + y^2}$ (x,y) to (2,0) but $y = \sqrt{x}$ so dist $= \sqrt{(x-2)^2 + (\sqrt{x})^2}$

$= \sqrt{x^2 - 3x + 4}$

We can maximize distance by maximizing radicand

So Maximize $r(x) = x^2 - 3x + 4$ $0 \le x \le 9$

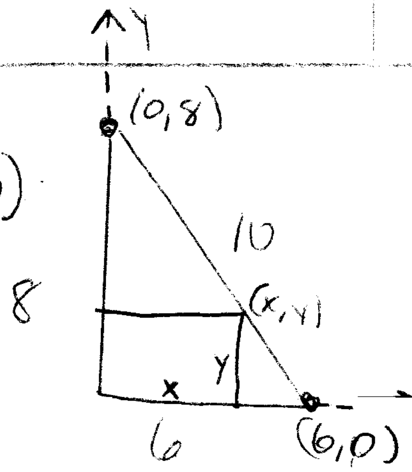
crit # $r'(x) = 2x - 3$

x	$3/2$	0	9
$r(x)$	$7/4$	4	58 ← max r

Find point: $x = 9 \Rightarrow (9,3)$ farthest point

Also Min $x = 3/2$ $\left(\frac{3}{2}, \sqrt{3/2}\right)$ closest point

10)



maximize area rectangle: $A = xy$
Need to relate x and y.
Can use similar Δs or super impose
Coordinate system and find eqn of line

$y = -\frac{4}{3}x + 8$

So $A(x) = x\left(-\frac{4}{3}x+8\right) = 8x - \frac{4}{3}x^2 \quad 0 \le x \le 6$

$A'(x) = 8 - \frac{8}{3}x \overset{set}{=} 0 \quad x = 3$

6

dimensions: $x = 3$

$y = 4$

x	3	0	6
$A(x)$	12	0	0

↖ MAX

Graph Practice
a) pg235 b) pg247 c) pg208

(11) To show exactly one real root must show
1) At least one AND 2) Not more than 1

Since $f(x) = x^3 + x - 1$ is conts and since $f(0) = -1 < 0$ then $f(1) = 1 > 0$ by IVT there is a $c \in (0,1)$ such that $f(c) = 0$.

Suppose f has two roots x_1 and x_2 so $f(x_1) = f(x_2) = 0$. Since f is conts + diffable Rolles Thm applies on (x_1, x_2). So there is a c such that $f'(c) = 0$. But $f'(c) = 3c^2 + 1$ which is never zero for c real. Contradiction. Thus f can not have more than one ~~zero~~

$\therefore f(x) = 0$ has exactly one real root

(12) Let $f(x) = x^{1/3}$. We want to approximate $f(8.03)$.

$f'(x) = \frac{1}{3x^{2/3}}$

Using $L(x)$

$L(x) = f(a) + f'(a)(x-a), \; a = 8$

$L(x) = 2 + \frac{1}{12}(x-8)$

$\sqrt[3]{8.03} \approx L(8.03) = 2 + \frac{1}{12}(8.03 - 8)$

$= 2 + \frac{1}{12}(.03)$

$= 2.0025$

OR Using differential;

$f(x + \Delta x) = f(x) + \Delta y \approx f(x) + dy$

$\sqrt[3]{8.03} = f(8 + .03) \approx f(8) + dy$

$= 2 + f'(8)\Delta x$

$= 2 + \frac{1}{12}(.03) = 2.0025$