# 6.4 Work

## Constant Force

When a body moves a distance d along a straight line as a result of being acted on by a force of constant magnitude F in the direction of motion, we define the work W done by the force on the body with the formula

$$W=Fd$$

In the SI system, units of force are kg-m/sec$^2$ or Newtons (N). The units of work are N-m or Joules.
In the British System, units of force are pounds(lbs). The units of work are ft-lbs.

Ex.

(b) How much work is done if a constant force of 50-lb is used to pull a cart 25 ft?_____

(a) How much work is done lifting a 20 kg box 2 meters off the ground? _____

## Variable Force

Suppose a particle moves along the x-axis from a to b acted upon by a continuous, variable force f(x).

Ex. 2 pg 371. When a particle is located a distance x feet from the origin, a force of $x^2+2x$ pounds acts on it. How much work is done in moving it from x=1 to x=3?

Ex HOOKE"S LAW for SPRINGS

A spring has a natural length of 20 cm. A 40 N force is required to stretch (and hold the spring) to a length of 30 cm. How much work is done in stretching the spring from 35 cm to 38 cm??

Hookw's Law states that the force required to maintain a spring stretched x units beyond its natural length is proportional to x: f(x) = kx

Ex: A 5-lb bucket is lifted from the ground into the air by pulling in 20 feet of rope at a constant speed. The rope weighs 0.08 lb/ft. How much work was spent lifting the bucket and rope?

Suppose that the bucket is leaking. It starts with 2 gallons (16 lb) of water in it and leaks at a constant rate. It finishes draining just as it reaches the top. How much work was spent lifting the water alone (neglect the rope and bucket.)

## Pumping liquids from containers

How much work does it take to pump all or part of the liquid from a container? To find out, we imagine lifting the liquid out one thin horizontal slab at a time and applying the equation W=Fd to each slab. If we sum these numbers let the number of slabs $\to \infty$ we obtain an integral.

Ex: A cylindrical tank of radius 3 ft and height 10 ft. is full of water weighing 62.5 lb/ft$^3$. How much work is done in emptying the tank by pumping water over the top? (b) out the bottom? (c) What if the tank is initially only ½ full?

Ex. 5 pg 372: (Set up differently than in book) A tank has the shape of an inverted circular cone with height 10 m and base radius 4 m. It is filled with water to a height of 8 m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (the density of water is 1000 kg/m$^3$)