MATH 131 -SAMPLE FINAL EXAM

(1) The solution to the inequality $-7x + 3 < 2$ is $(1/7, \infty)$ $-7x < -1$ $x > 1/7$

(2) $(-64)^{\frac{2}{3}}$ = 16 (answer should be an integer)

(3) The least common denominator of $\frac{1}{x^2y^3}, \frac{7}{4x^5y}, \frac{8}{5x^6}$ is $20x^6y^3$

(4) Expand. (Rewrite as a sum of difference of multiples of logarithms): $\log\left(\frac{x^2}{y^3z^5}\right)$ $2\log x - 3\log y - 5\log z$

(5) The domain of the function $f(x) = \frac{3}{x-6}$ is $\{x \mid x \neq 6\}$

(6) The slope of the line through the points (5, 2) and (-3, 7) is $-\frac{5}{8}$

(7) The graph of $y - 2 = -3(x + 5)^2$ is a parabola with vertex $(-5, 2)$

(8) Factoring, $2x^3 - 54 =$ $2(x-3)(x^2+3x+9)$

(9) If the legs of a right triangle are 3 cm and 6 cm long, the length of the hypotenuse is $\sqrt{45} = 3\sqrt{5}$

(10) $\sqrt[3]{432} =$ $6\sqrt[3]{2}$ $\sqrt[3]{8 \cdot 54} = \sqrt[3]{8 \cdot 27 \cdot 2}$

CIRCLE T FOR TRUE, F FOR FALSE.

T F (11) $\sqrt{9} = \pm 3$

T F (12) $(x + y)^3 = x^3 + y^3$.

T F (13) Rationalizing the denominator, $\frac{24}{\sqrt{8}} = 6\sqrt{2}$ $\frac{24}{2\sqrt{2}} = \frac{12\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{12\sqrt{2}}{2}$

T F (14) $\left(3x^2y\right)^0 = 0$ It's 1

T F (15) The slope of a line perpendicular to 4x+6y=9 is 3/2.

(16) Solve: (a) $3x(x+5) = 2$ $3x^2 + 15x - 2 = 0$ $x = \frac{-15 \pm \sqrt{249}}{6}$ (b) $x^2 = \frac{2}{3}x + \frac{4}{9}$ $9x^2 = 6x + 4$ $9x^2 - 6x - 4 = 0$ $x = \frac{1 \pm \sqrt{5}}{3}$

(17) Simplify. Do not leave negative exponents.

(a) $(2xy^3)(5x^3y^{-3})(7x)$ $70x^5$ (b) $\left(-5a^{-7}b^3\right)^{-3}$ $\frac{a^{21}}{-125b^9}$ (c) $\frac{-8r^2s^{-7}t}{16r^{-9}s^{-7}t^3}$ $\frac{-r^{11}}{2t^2}$

(18) Solve : (a) $\log_2(x+1) - \log_2(x) = 3$ $\log_2\left(\frac{x+1}{x}\right) = 3$ $2^3 = \frac{x+1}{x}$ $x = 1/7$ (b) $3^{4-x} = 7$ $\longrightarrow$ $\log 3^{4-x} = \log 7$ $(4-x)\log 3 = \log 7$ $x = 4 - \frac{\log 7}{\log 3}$

(19) Given $g(x) = \frac{1}{x-3}$ and $f(x) = x^3$, find and simplify the following

a) $f(-3) =$ -27 b) $g\left(\frac{1}{x}\right) =$ $\frac{1}{\frac{1}{x} - 3} = \frac{x}{1-3x}$ c) $(g \circ f)(x) =$ $\frac{1}{x^3 - 3}$

(13) SIMPLIFY EACH OF THE FOLLOWING EXPRESSIONS

(a) $\frac{2}{x-1} + \frac{2x}{1-x^2} - \frac{x}{x^2+2x+1}$

$\frac{2}{x-1} - \frac{2x}{x^2-1} - \frac{x}{(x+1)^2}$

$\frac{2}{x-1} - \frac{2x}{(x-1)(x+1)} - \frac{x}{(x+1)^2}$

$\frac{2(x+1)^2 - 2x(x+1) - x(x-1)}{(x-1)(x+1)^2} = \cdots = \frac{-x^2+3x+2}{(x-1)(x+1)^2}$

(b) $\frac{5x^2 - 14xy - 3y^2}{y^2 + 4xy - 5x^2} \bullet \frac{8y - 2x}{4y^2 + 3xy - x^2} + \frac{7x^2 - 20xy - 3y^2}{y^2 - x^2}$ divide

$\frac{(5x+y)(x-3y)}{(y-x)(y+5x)} \cdot \frac{2(4y-x)}{(4y-x)(y+x)} \cdot \frac{(y-x)(y+x)}{(7x+y)(x-3y)}$

$\frac{2}{7x+y}$

$\sqrt[3]{\dfrac{x^6}{4y^2}} \cdot \dfrac{\sqrt[3]{2y}}{\sqrt[3]{2y}}$

(c) $\dfrac{\sqrt[3]{3x^7y^2}}{\sqrt[3]{12xy^4}} = \dfrac{x^2\sqrt[3]{2y}}{2y}$

(d) $\sqrt[4]{8x^2y^7}\ \sqrt[4]{6x^5y^5} = 2xy^3\sqrt[4]{3x^3}$ $\quad \sqrt[4]{48x^7y^{12}}$ $\quad \sqrt[4]{16x^4y^{12}\ 3x^3}$

(e) $\left(-12a^5b^{-\frac{1}{2}}\right)^2\left(a^{-5}b^{5/4}\right)^4 = \dfrac{144b^4}{a^{10}}$

$144a^{10}b^{-1}a^{-20}b^5$

(f) $\dfrac{x^{-1}+y^{-2}}{x^{-2}-y^{-1}} =$ ______

$\dfrac{\frac{1}{x}+\frac{1}{y^2}}{\frac{1}{x^2}-\frac{1}{y}} \cdot \dfrac{x^2y^2}{x^2y^2} = \dfrac{xy^2+x^2}{y^2-x^2y} = \dfrac{x(y^2+x)}{y(y-x^2)}$

(20) Solve the system

$$\begin{cases} 2x-2y+z=-2 \\ x+y-3z=3 \\ x-3y+z=-5 \end{cases}$$

$(1,2,0)$

(21) If $f(x) = \dfrac{2x-3}{4}$, find $f^{-1}(x)$ and show $f[f^{-1}(x)] = x$. $\quad y=\dfrac{2x-3}{4}$ Switch $x=\dfrac{2y-3}{4} \Rightarrow f^{-1}(x)=\dfrac{4x+3}{2}$

$f(f^{-1}(x)) = f\left(\dfrac{4x+3}{2}\right) = \dfrac{2\left(\frac{4x+3}{2}\right)-3}{4} =$

(22) Solve: $\dfrac{3x-1}{x^2+5x-14} = \dfrac{1}{x-2} - \dfrac{2}{x+7}$ $\quad x=3$

(23) On the first part of a trip to Maine, traveling on freeway, Mary averaged 60 miles per hour. On the rest of the trip, which was 10 miles longer than the first part, she averaged 50 miles per hour. Find the distance to Maine if the first part of the trip took one-half hour less time than the second part. (SHOW SET UP AND SOLUTION)

(24) Factor completely:

(a) $12 + 4x - 3x^2 - x^3$ $\quad (3+x)(2-x)(2+x)$

(b) $16x^2 + 28xy + 6y^2$ $\quad 2(4x+y)(2x+3y)$

(c) x^4-81 $\quad (x^2+9)(x-3)(x+3)$

(d) $x^2 + 2x - y^2 - 2y$ $\quad (x-y)(x+y+2)$

hint: rearrange

(25) Find the equation of the line which passes through (2, 5) and which is perpendicular to the line 3x - 5y = 8. $\quad y=-\frac{5}{3}x+\frac{25}{3}$

(26) To meet a sales quota, a car salesman must sell 22 new cars. He must sell 4 times more small cars than large cars, and 4 more mid-sized cars than large cars. How many of each size car should he sell? (SET UP A SYSTEM OF EQUATION AND SOLVE)

S: # small, m = # med

L = # large

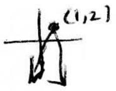
S+m+L=22

S=4L

m=L+4

$\Rightarrow L=3,\ S=12,\ m=7$

(27) Sketch the graph of $f(x) = -3x^2 + 6x - 1.$ $= -3(x-1)^2+2$

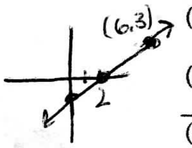


(28) Solve: $\sqrt{2x+4} - \sqrt{x+3} = 1$ $\Rightarrow (\sqrt{2x+4})^2 = (\sqrt{x+3}+1)^2$

$2x+4 = x+3+2\sqrt{x+3}+1$

$x = 2\sqrt{x+3} \Rightarrow x^2=4(x+3) \Rightarrow \boxed{x=6}, \cancel{-2}$

(29) Sketch the graph: $3x - 4y = 6$



(30) Solve the following inequalities and express your answer using INTERVAL notation.*(10 points each)*

(a) $|3x+5|>2$

$3x+5<-2$ or $3x+5>2$

$3x<-7 \qquad 3x>-3$

$x<-7/3 \qquad x>-1$

$(-\infty,-7/3)\cup(-1,\infty)$

(b) $\dfrac{x-4}{2x+1} \le 0$ Signchart

$+ \quad - \quad +$

$-1/2 \qquad 4$

$(-1/2, 4]$