MATH 10 - TEST 1 **(CHAPTER 1 and 2)** **100 points** **Spring 2012** **NAME:**________________________ **YOU MUST SHOW YOUR WORK. PRESENTATION COUNTS!** **Phones must be OFF and put away. No graphing calculators allowed. No scratch paper allowed.**

CIRCLE T FOR TRUE, F FOR FALSE. *(3 points each)*

T F (1) For any nxn matrices A, B, and C, if AB = AC, then B = C.

T F (2) If A and B are invertible nxn matrices, then AB is invertible, and $(AB)^{-1} = B^{-1} A^{-1}$.

T F (3) $-3R_1 + R_2 \rightarrow R_1$ is an elementary row operation..

T F (4) The associative law for multiplication holds true for matrix multiplication.

T F (5) If A and B are square matrices such that AB = 0 and B is invertible, then A = 0.

T F (6) If A is invertible then the system AX = 0 has infinitely many solutions.

T F (7) If A is row equivalent to B and A is invertible then B is invertible.

T F (8) If AB=I then B is the inverse of A.

SHOW ALL WORK NEATLY AND PUT <u>BOX</u> AROUND ALL ANSWERS.

(9) Compute $\begin{vmatrix} 3 & -1 & 2 & 0 \\ -2 & -3 & 1 & 3 \\ 0 & -1 & 4 & 1 \\ 5 & 0 & -2 & 3 \end{vmatrix}$. *(8 points)*

(10) Given the matrices: $A = \begin{bmatrix} 0 & 0 & -3 \\ 1 & 3 & 3 \\ 1 & 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 4 & 3 \\ 1 & 3 & 3 \\ 1 & 2 & 3 \end{bmatrix}$

a) Compute each of the following. You may use properties from class to shorten your work, but make it clear what you are doing..

i) the second row of AB. *(4 points)*

ii) B^{-1} *(10 points)*

iii) $\left(B^T\right)^{-1}$ *(3 points)*

iv) $\det\left(A^{-1}\right)$ *(3 points)*

b) Express A in the form A = EB, where E is an elementary matrix. *(6 points)*

(11) Use matrix methods (Gaussian elimination or Gauss Jordan) to solve: *(10 points)*

$2x + y + z = 16$
$-x - 2y - z = -3$
$x + y + 2z = 9$

You must obtain row (or reduced row) echelon form. Be sure to label operations performed at each step.

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(12) Given an nxn matrix A, we have found 7 equivalent statements in "the big theorem". Name four of them. *(8 points)*

(13) If A is a symmetric nxn matrix and B is any nxm matrix, prove that $B^T AB$ is an mxm symmetric matrix., *(6 points)*

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(14) Prove: The matrix A= $\begin{bmatrix} 1 & a & b \\ -a & 1 & c \\ -b & -c & 1 \end{bmatrix}$ is invertible.

(6 points)

(15) Determine all values of B and C for which the system *(12 points)*

$x - y + z = 2$
$x + y - z = C$
$2x - y + Bz = -1$

has a) a unique solution ________________

b) infinitely many solutions ______________

c) no solution ______________________